A Mathematical Analysis of the COVID-19 SEIQRS Model Incorporating the Effect of Precautionary Measures

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Abstract

This study considers the dynamics of an infectious disease through an epidemic SEIQRS model and also discusses the stability of both the endemic and disease-free equilibrium. The basic reproduction number is determined using the next generation method. The effects of quarantine (or isolation) strategies to control the spread of the disease will be examined. This study shows that the quarantine (or isolation) of susceptible, exposed and infected people can play an important role in preventing the further spread of the disease.

2020 Mathematical Subject Classifications: 49K15, 65L05, 34D20, 34D23, 92B05, 92D05.

Keywords and phrases: Basic reproduction number, Routh-Hurwitz criteria, Jacobian matrix, Disease-free point, Endemic point, Lyapunov method etc.

1. Introduction

A contagious disease induced by a newly detected virus can threaten the human as we see in the case of Coronavirus. To stop and downturn the transmission, one should be well-educated about the virus, the disease causes, and the mechanics of its spreads. When a contagious person coughs or sneezes, the virus spreads through their droplets of saliva or release from their nose. Many clinical prosecutions are investigating possible treatments [7].

The outbreak has been announced an epidemic, under the epidemic diseases act, 1897. Including the educational institutions and many businesses have been abandon after the declaration of lockdown by the government.

Mathematical modeling is used to explain the dynamics of a contagious disease. Here, we propose a deterministic SEIQRS (susceptible, exposed, infected, quarantined, recovered, susceptible) model to try to clarify the mechanism of the infectious disease.

With the help of classic susceptible, infected, removed (SIR) model developed by Kermack and McKendrick [28] or the SEIR model (Here E represents the exposed population) developed by Rvachev and Longini [12], some epidemic models have been specified. To be better prepared against future catastrophic epidemics, In 2007, Yan et al. [29] explored quarantine and separateness methods and establish that these are the most effective specifications in the SARS (Severe acute respiratory syndrome) outbreak control study. In 2009, Tang et al. [9] proposed a SEIQR model to evaluate its mathematical possession, and define a Differential Evolution (DE) method for the framework designation of the sample. In his investigation, the SEIR model executed by Massad et al. [4] is expanded by a new category of quarantine people and a new proportion at which infected people shift to the quarantine category. In the same year, Gerberry and Milnar [3] proposed and studied a SEIQR model for childhood infections. They expand the work of Feng et al. from a SIQR model to a SEIQR model. In 2010, Mishra and Jha [1] formulated a SEIQRS figure for the communication of destructive articles in a computer system. Soni et al. [26] used numerical and statistical method to study the uncertainty in the spread of COVID-19. Sharma et al. [23] proposed an SEIR epidemic model with a saturated incidence rate and investigated the existence and stability of equilibriums of the epidemic model. They [22] analyzed an SIRS model with asymptotically homogeneous transmission function and emigration rate. Sharma S. and Sharma P.K. [21] consider an SIQR model with a Holling type-II incidence rate. D Pal et al. [17] consider an SEQIR model to analyze the situation in the five states of India. Singh, P., and Gupta, A. [24] proposed an Generalized SIR(GSIR) epidemic model to for the predictive monitoring of pandemic COVID-19. HM Youssef, N Alghamdi, MA Ezzat [30] proposed a modified SEIQR model to study the spread infection in Saudi Arabia.

In this paper, an SEIQRS model is proposed to analyze the epidemic stability of the dynamic system at the equilibrium point of disease-free and endemic disease. The basis reproduction number R_0 is estimated by the next generation matrix technique. Moreover, the validity of our model is checked by numerical simulation. The paper is arranged as follows: First of all, 'Introduction' is discussed in the first section. Furthermore, under some assumption, a mathematical model is formulated in the second section. The model properties are discussed in the third section. The basic reproduction number R_0 and stability points are discovered in the fourth section. The model stability at equilibrium points are interpreted in the fifth section. Simulation is performed by taking appropriate parameter values to check the validity of the system model.

2.Formulation of the Model

2.1Diagram of the Model

In this section, we formulate the model under some assumption. Model diagram give a clear picture of the dynamics of the disease. We describe the various classes and model parameters. Our SEIQRS model is formulated by the system of differential equations.

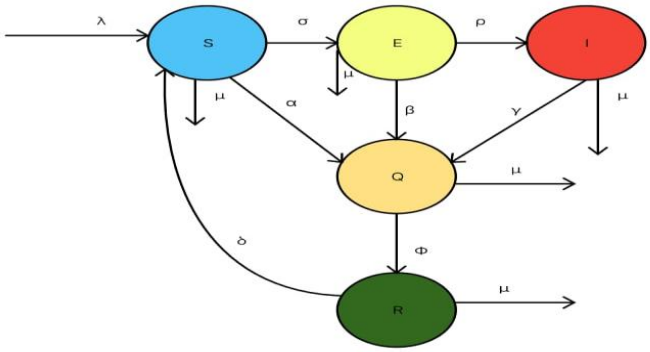


Figure 2.1 SEIQRS Model

2.2Assumptions

We construct a SEIQRS model (See Figure 2.1) with the following assumption:

1. Population mixing is homogeneous.

2. Recovered individuals may be susceptible again after losing immunity.

3. Susceptible individuals become contaminated only when they come in the connection with an infected individual directly (person to person) or indirectly (Through droplets of the virus).

4. During the incubation period, infected individuals could not infect others.

5. During the quarantine/isolation period infected or exposed could not infect others and after gaining strong immunity all quarantined/isolated individuals enter into the recovered class.

6. It is considered that all migrants/births enter into the susceptible class only.

The five categories S, E, I, Q, R , 9 model parameters and their corresponding inverse model parameters of the model SEIQRS (In Figure 2.1) are interpreted in Table 2.1, Table 2.2 and Table 2.3 respectively.

Table 2.1 Classes

Susceptible	*S*	Susceptible individuals to sickness who can acquire the infection if they are uncovered to it.
Exposed	*E*	Exposed people to disease who are infected but have not yet come to be infectious and cannot transmit the infection to other susceptible people.
Infected	*I*	People contaminated by disease and eligible for transmitting the disease to the 'S' class people.
Quarantine	*Q*	Quarantined people who cannot contract the disease through infected individuals.
Recovered	*R*	Recovered people recovered and protected from the disease temporarily.

Table 2.2 Explanation of model parameters.

Parameter	Name	Units	Significance
λ	Birth rate	In days^{-1}	The rate of natural birth per 1,000 populations per year.
α	Dispatch rate (*S* class to Quarantine class)		The rate at which *S* class people become quarantine people.
β	Dispatch rate (*E* class to Quarantine class)		The rate at which exposed people transmits to quarantine people.
γ	Dispatch rate (Infected to Quarantine)		The rate at which infected people transmits quarantine people.
δ	Dispatch rate (Recovered to Susceptible)		The rate at which recovered people become susceptible individuals again by losing their immunity.
ρ	Incubation rate (Exposed to Infected)		The rate at which exposed people become infected by incubating infection.
Φ	Recovery rate (Quarantine to Recovered)		The rate at which quarantined people become recovered by temporary or permanent immunity.
σ	Dispatch rate (S class to Exposed)		The rate at which susceptible people become exposed people.
μ	Death rate	Number of deaths per 1000 per year	Natural death rate in each class per year.

Table 2.3 Explanation of inverse model parameters.

Parameter	Term	Units	Explanation
λ^{-1}	Mean immigrant duration	In days	Time taken by a immigrant to be a susceptible.
α^{-1}	Mean susceptible-quarantine duration		Time taken by a susceptible to be quarantine.
β^{-1}	Mean exposed-quarantine duration		Time taken by an exposed to be quarantine.
γ^{-1}	Mean infected-quarantine duration		Time taken by an infected to be quarantine.
σ^{-1}	Mean susceptibility duration		Time taken by a susceptible to be an exposed.
ρ^{-1}	Mean latency duration		Time taken by an exposed to be an infected.
δ^{-1}	Mean immunity duration		Time taken by a recovered to be a susceptible.
Φ^{-1}	Mean quarantine duration		Time taken by quarantine to be a recovered.

2.3 Mathematical form of the Model

Mathematically, the SEIQRS model (In Figure 2.1) is reflected as following:

$$(2.1)\ \frac{dS}{dt} = \lambda N - \frac{\sigma SI}{N} - \mu S - \alpha S + \delta R,$$

$$\frac{dE}{dt} = \frac{\sigma SI}{N} - \mu E - \rho E - \beta E,$$

$$\frac{dI}{dt} = \rho E - \mu I - \gamma I,$$

$$\frac{dQ}{dt} = \alpha S + \beta E + \gamma I - \mu Q - \Phi Q,$$

$$\frac{dR}{dt} = \Phi Q - \mu R - \delta R,$$

Where $\lambda, \sigma, \rho, \mu, \alpha, \beta, \gamma, \Phi, \delta$ all are non negative constant parameters.

With the whole population,

(2.2) $N(t) = S(t) + E(t) + I(t) + Q(t) + R(t)$,

Where S, E, I, Q, R represent the people in different corresponding classes on particular time‘t’.

The entire people are operated by the following equation which obtained by adding the equations of the above system (2.1):

$$\frac{d(S + E + I + Q + R)}{dt} = \frac{dN}{dt} = \lambda N - \mu(S + E + I + Q + R) = N(\lambda - \mu),$$

using separation of variables method and after integrating, we get

$$N(t) = N_0 e^{(\lambda - \mu)t} .$$

Time-varying population *N(t).*Where$N_0$is the initial whole population at the time *t=0*.

2.4Transformation without Dimension

We apply the method used by ([2],[10], [11], [14],[18], [19], [20], [27]) and change the scale of each class population by the entire population to streamline the calculation.

We use the transformations:

$$s = \frac{S}{N}, \quad e = \frac{E}{N}, \quad i = \frac{I}{N}, \quad q = \frac{Q}{N}, \quad r = \frac{R}{N}.$$

And apply it to the system (2.1).

Where *s, e, i, q, r* indicate the fractions of the people in their corresponding classes respectively.

$$s + e + i + q + r = 1.$$

The transformed system is formulated as:

(2.3) $\frac{ds}{dt} = \lambda - \sigma si + \delta r - (\alpha + \mu)s,$

$$\frac{de}{dt} = \sigma si - (\beta + \rho + \mu)e,$$

$$\frac{di}{dt} = \rho e - (\gamma + \mu)i,$$

$$\frac{dq}{dt} = \alpha s + \beta e + \gamma i - (\Phi + \mu)q,$$

$$\frac{dr}{dt} = \Phi q - (\delta + \mu)r,$$

(Reduced system)

$$(2.4)\frac{ds}{dt} = A - \sigma si + \delta r - Bs$$

$$\frac{de}{dt} = \sigma si - Ce$$

$$\frac{di}{dt} = \rho e - Di,$$

$$\frac{dq}{dt} = \alpha s + \beta e + \gamma i - Eq,$$

$$\frac{dr}{dt} = \Phi q - Fr,$$

Where,

$$A = \lambda, B = \alpha + \mu, C = \beta + \rho + \mu, D = \gamma + \mu, E = \Phi + \mu, F = \delta + \mu,$$

are non-negative constants.

3.Fundamental Properties of the Model

In this section, we discuss the positivity of the solution of the system, equilibrium points of the system, and the basic reproduction number of the disease.

3.1Invariant Region

Since this research is on the human population, therefore variables remain positive always. Accordingly, the system (2.4) in the region Ω is positively constraint always. Where

$$\Omega = \{(s, e, i, q, r) \in R_+^5 > 0, s + e + q + i + r \leq 1, s > 0, e > 0, i > 0, q > 0, r > 0)$$

The feasible region is positively invariant and all the solutions are bounded and belong to Ω.

3.2Equilibrium Points and the basic reproduction number R_0

To compute R_0, the next-generation matrix method given by ([15], [16]) is used. The R_0 is the mean number of secondary contagious cases which would be generate in a totally susceptible population by one infected patient. Soni et al. [25] find the basic reproduction number and their corresponding herd immunity by a review study.

Our model's disease free equilibrium $(DFE) x_{DFE} = (s_0, 0, 0, 0, 0) = \left(\frac{A}{B}, 0, 0, 0, 0\right)$ and endemic equilibrium (EE) $x_{EE} = (s^*,\ e^*,\ i^*,\ q^*, r^*)$. Now, let

$$x = (E, I, Q, S, R)^T$$

The reduced model is,

$$\frac{dx}{dt} = F(x) - V(x),$$

where,

$$F(x) = (\sigma si, 0, 0, 0, 0)^T$$

$$V(x) = (Ce, -\rho e + Di, -\alpha s - \beta e - \gamma i + Eq, -A + \sigma si + Bs - \delta r, -\Phi q + Fr)^T$$

Now we can get,

$$F = \begin{pmatrix} 0 & \sigma s \\ 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} C & 0 \\ -\rho & D \end{pmatrix},$$

$$\Rightarrow FV^{-1} = \frac{1}{CD}\begin{pmatrix} \sigma\rho s & \sigma sC \\ 0 & 0 \end{pmatrix},$$

The spectral radius of $\rho(FV^{-1})$ is equal to R_0.

$$R_0 = \frac{\sigma\rho s}{CD}$$

$$R_0 = \frac{\sigma\rho A}{BCD}$$

(3.1) $$R_0 = \frac{\sigma\rho\lambda}{(\alpha+\mu)(\beta+\rho+\mu)(\gamma+\mu)}.$$

Since at disease free equilibrium, $s_0 = \frac{A}{B} = \frac{\lambda}{\alpha+\mu}$.

For *DFE*, we take, $\frac{ds}{dt} = 0$ and $e_0 = 0, i_0 = 0, q_0 = 0, r_0 = 0$.

$$\frac{\mathrm{ds}}{\mathrm{dt}} = \mathrm{A} - \sigma \mathrm{s}_0 \mathrm{i}_0 + \delta \mathrm{r}_0 - \mathrm{Bs}_0 = 0$$

$$\Rightarrow s_0 = \frac{A}{B} = \frac{\lambda}{\alpha + \mu}.$$

For *EE*, we take,

$$\frac{ds}{dt} = 0, \quad \frac{de}{dt} = 0, \quad \frac{di}{dt} = 0, \quad \frac{dq}{dt} = 0, \quad \frac{dr}{dt} = 0.$$

Therefore, we get,

$$s^* = \frac{A}{BR_0}, e^* = \frac{i^* D}{\rho},$$

$$i^* = \frac{\frac{A}{R_0}\left\{\frac{\Phi\alpha}{BEF} + \frac{R_0 - 1}{\delta}\right\}}{\left\{\frac{\sigma A}{\delta B R_0} - \frac{(BD + \rho\gamma)\Phi}{\rho EF}\right\}}.$$

$$q^* = \frac{1}{E}\left[\alpha s^* + \left(\frac{\beta D}{\rho} + \gamma\right) i^*\right] = \frac{1}{E}\left[\frac{\alpha A}{BR_0} + \left(\frac{\beta D}{\rho} + \gamma\right) i^*\right],$$

$$r^* = \frac{\Phi}{EF}\left[\alpha s^* + \left(\frac{\beta D}{\rho} + \gamma\right) i^*\right] = \frac{\Phi}{EF}\left[\frac{\alpha A}{BR_0} + \left(\frac{\beta D}{\rho} + \gamma\right) i^*\right].$$

4. Stability Analyses at the Equilibrium Point

In this section, we discuss about the local and global stability of the system at the disease-free and endemic equilibrium points.

4.1 Local Stability at the Point x_{DFE}

Theorem* 4.1If* $R_0 \leq 1$, *then* x_{DFE} *is asymptotically locally stable, otherwise unstable.*

Proof Within this model, the stability of equilibrium point locally, is deduced from the Jacobean matrix. The Jacobean matrix is,

$$J(x) = J(s, e, i, q, r) = \begin{pmatrix} -B - \sigma i & 0 & -\sigma s & 0 & 0 \\ \sigma i & -C & \sigma s & 0 & 0 \\ 0 & \rho & -D & 0 & 0 \\ \alpha & \beta & \gamma & -E & 0 \\ 0 & 0 & 0 & \Phi & -F \end{pmatrix}.$$

At *DFE*, the characteristic equation matrix is given by

$$|J(x_{DFE}) - \lambda I| = \begin{vmatrix} -B-\lambda & 0 & 0 & -\frac{\sigma A}{B} & 0 \\ 0 & -C-\lambda & \frac{\sigma A}{B} & 0 & 0 \\ 0 & \rho & -D-\lambda & 0 & 0 \\ \alpha & \beta & \gamma & -E-\lambda & 0 \\ 0 & 0 & 0 & \Phi & -F-\lambda \end{vmatrix}$$

$$(4.1)|J(x_{DFE} - \lambda I)| = (-F-\lambda)(-E-\lambda)(-B-\lambda)[(-C-\lambda)(-D-\lambda) - \frac{\sigma\rho A}{B} = 0$$

The eigen values are given by,

$$\lambda_1 = -F,\ \lambda_2 = -E,\ \lambda_3 = -B\ and\ (C+\lambda)(D+\lambda) - \frac{\sigma\rho A}{B} = 0$$

By applying the Routh-Hurwitz criteria of second degree polynomial, on the polynomial

$$\lambda^2 + (C+D)\lambda + CD - \frac{\sigma\rho A}{B} = 0$$

The remaining two roots will be contain negative real parts if

$C + D > 0$ (which is true obviousely), $CD - \frac{\sigma\rho A}{B} > 0(\text{or})R_0 < 1.$

Therefore, by the Routh-Hurwitz standards of fifth degree polynomial, we conclude that $R_0 < 1$,if and only if all the eigen values have negative real parts. Hence, the *DFE* is stable locally asymptotically if $R_0 \leq 1$ and unstable locally if, $R_0 > 1$. This concludes the proof.

4.2Global Stability at the Point x_{DFE}

Theorem* 4.2The disease-free equilibrium point x_{DFE} is asymptotically stable globally if $R_0 < 1$.*

ProofTo check the stability on *DFE* globally, we construct a suitable Lyapunov function,

(4.2) $L = \rho e + (\beta + \rho + \mu)i$.

Clearly, L is positive definite. On differentiation equation (4.2), we obtain,

$$L' = \rho e' + (\beta + \rho + \mu)i'$$

$$\Rightarrow L' = \rho\{\sigma si - (\beta + \rho + \mu)e\} + (\beta + \rho + \mu)\{\rho e - (\gamma + \mu)i$$

$$\Rightarrow L' = \{\rho\sigma s - (\beta + \rho + \mu)(\gamma + \mu)\}i$$

$$\Rightarrow L' = \{(\beta + \rho + \mu)(\gamma + \mu)\}\left\{\frac{\rho\sigma s}{(\beta + \rho + \mu)(\gamma + \mu)} - 1\right\}i$$

$$\Rightarrow L^{'} = \{(\beta+\rho+\mu)(\gamma+\mu)\}\left\{\frac{\sigma\rho A}{B(\beta+\rho+\mu)(\gamma+\mu)} - 1\right\} i$$

$$\Rightarrow L^{'} = \{(\beta+\rho+\mu)(\gamma+\mu)\}\{R_0 - 1\} i$$

Clearly, $L' < 0$ if and only if $R_0 < 1$. Also $L' = 0$ if and only if $R_0 = 1$ or $i = 0$. So by the LaSalle's invariance principle [8], x_{DFE} is stable asymptotically globally with $R_0 < 1$. This completes the proof.

4.3Local Stability at the Point x_{EE}

Theorem* 4.3The point x_{EE} is asymptotically stable locally, if $R_0 > 1$.*

Proof In *EE*, the eigen equation of matrix is provided by,

$$|J(x_{EE}) - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -B-\sigma i^* - \lambda & 0 & -\sigma s^* & 0 & 0 \\ \sigma s^* & -C-\lambda & \sigma s^* & 0 & 0 \\ 0 & \rho & -D-\lambda & 0 & 0 \\ \alpha & \beta & \gamma & -E-\lambda & 0 \\ 0 & 0 & 0 & \Phi & -F-\lambda \end{vmatrix} = 0.$$

We have the eigen values, $\lambda_1 = -F,\ \lambda_2 = -E.$

And the other three eigen values will be determined by,

$$(4.3)\ \lambda^3 + (B+C+D+\sigma i^*)\lambda^2 + (B+\sigma i^*)(C+D)\,\lambda + \frac{C^2D^2}{\rho} = 0.$$

Again, for applying the Routh-Hurwitz criteria on the cubic equation,

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0.$$

We must have, $(i) a_1 > 0,\ (ii) a_3 > 0,\ (iii)\ a_1 a_2 > a_3.$

On comparison, we get,

$$(i)\ a_1 = B + C + D + \sigma i^* > 0$$

$$(ii) a_3 = \frac{C^2D^2}{\rho} > 0$$

$$(iii) a_1 a_2 > a_3,$$

$$\Rightarrow (B+C+D+\sigma i^*)(BC + BD + \sigma C i^* + \sigma D i^*) > \frac{C^2D^2}{\rho}$$

$$\Rightarrow \frac{(B+C+D+\sigma i^*)(B+\sigma i^*)(C+D)\rho}{C^2D^2} > 1$$

(i), (ii) and (iii) conditions will be satisfied if $i^* \geq 0$,

where,

$$(4.4)\ i^* = \frac{\frac{1}{\delta}\left\{A - \frac{BCD}{\sigma\rho}\right\} - \frac{\alpha\phi CD}{\sigma\rho EF}}{\left\{\frac{CD}{\sigma\rho} - \frac{(BD+\rho\gamma)\Phi}{\rho EF}\right\}}\ .$$

Thus, by the standard, decided by Routh-Hurwitz, all the eigen roots of the equation (4.3) have real components with negative sign for $R_0 > 1$. Hence the endemic equilibrium point x_{EE} is stable asymptotically locally. This completes the proof.

4.4Global Stability at the Pointx_{EE}

We use the geometric method of Li and Muldowney ([12], [13]). Described below briefly:

Let's suppose that the system is autonomous and dynamic,

$$(4.5)\ \dot{x} = f(x)$$

Where, $f: \Omega \rightarrow R^n, \Omega \subset R^n$ open set, and $f \in C^1(\Omega)$.

Some additional hypotheses are supposed:

(H1) Ω is the region which is simply connected,

(H2) ∃ a set $\Gamma \subset \Omega$which is compact and absorbing,

(H3) ∃ a unique stability point $\bar{x} \in \Omega$ of (4.5).

With the above assumptions, Li and Muldowney get the following results.

Theorem* 4.4With the above hypotheses (H1), (H2) and (H3), point $\bar{x} \in \Omega$ asymptotical stable globally in $lnt\ \Omega$ with $\bar{q}_2 < 0$, where*

$$(4.6)\bar{q}_2 = \lim_{t\to\infty} \sup_{x_0\in\Gamma} \frac{1}{t}\int_0^t v\left(B\left(x(s,x_0)\right)\right) ds.$$

Where the matrix $B = P_f P^{-1} + PJ^{[2]}$and in the path of solution of f, P_f is acquired by displacingP_i of P by its derivative and the system has $J^{[2]}$ as a additive matrix of order 2 which is compound.

Furthermore, The Lozinskiĭ measure is υ(B). Concerning a norm $||.||$ *in* R^n,

$$\upsilon(B) = \lim_{h\to 0^+} \frac{\left||I + hB|\right| - 1}{h} \quad . \tag{4.7}$$

Furthermore, for testing the stability globally on the x_{EE} point we prove the following lemma and theorem.

Lemma* 4.1In Int Ω system (*2.4*) is persistent uniformly, if* $R_0 > 1$.

ProofThe *Theorem* 4.2 indicates that if $R_0 < 1 \Rightarrow L' < 0$, and if $R_0 > 1 \Rightarrow L' > 0$ which directs to the fluctuation on x_{DFE}. Using the result of butler et al. [5]and Freedman et al. [6] and, we deduce that since x_{DFE} is uncertain when $R_0 > 1$, persistency and uniformly of system in *Int* $\Omega \Rightarrow$ ∃ a constant $c > 0$such that

$$\lim_{t\to\infty} s(t) > c, \quad \lim_{t\to\infty} e(t) > c, \quad \lim_{t\to\infty} i(t) > c, \quad \lim_{t\to\infty} q(t) > c, \quad \lim_{t\to\infty} r(t) > c\,.$$

With $\big(s(0), e(0), i(0), q(0), r(0)\big) \in \Omega$.

The boundedness of Ω with uniformly persistency identical to the presence of an absorbing compact set in *Int* Ω.

***Theorem* 4.5** x_{EE} *is stable globally asymptotically in Int*Ω *when* $R_0 > 1$ a*nd*$\text{ƀ} > 0$ *(*ƀ *is specified in the end of the theorem).*

Proof Our system (2.4) is endless uniformly in $Int\ \Omega$, whenever$R_0 > 1$, so ∃ a set$\Gamma \subset Int\ \Omega$ [27] which is absorbing and compact.

∵ The system (2.4) is uniformly persistent in $Int\ \Omega$,

∴ ∃ $m > 0$, free from the initial database in $Int\ \Omega$, thus all solutions $\big(s(t), e(t), i(t), q(t), r(t)\big) \in Int\ \Omega$of the system (2.4) fulfill.

$$\lim_{t\to\infty} Inf\ s(t) > m, \quad \lim_{t\to\infty} Inf\ e(t) > m, \quad \lim_{t\to\infty} Inf\ i(t) > m,$$
$$\lim_{t\to\infty} Inf\ q(t) > m, \lim_{t\to\infty} Inf\ r(t) > m\,.$$

Furnished that, $\big(s(0), e(0), i(0), q(0), r(0)\big) \in Int\ \Omega$.

Jacobean matrix of system (2.4) is given by,

$$J\ (s,e,I,q) = \begin{pmatrix} -(\alpha+\mu+\sigma i) & 0 & -\sigma s & 0 \\ \sigma i & -(\beta+\rho+\mu) & \sigma s & 0 \\ 0 & \rho & -(\gamma+\mu) & 0 \\ \alpha & \beta & \gamma & -(\Phi+\mu) \end{pmatrix}$$

(4.8) $$J(s,e,i,q) = \begin{pmatrix} -M_{11} & 0 & -\sigma s & 0 \\ \sigma i & -M_{22} & \sigma s & 0 \\ 0 & \rho & -M_{33} & 0 \\ \alpha & \beta & \gamma & -M_{44} \end{pmatrix},$$

where,

$$M_{11} = \alpha+\mu+\sigma i, M_{22} = \beta+\rho+\mu, M_{33} = \gamma+\mu, M_{44} = \Phi+\mu.$$

Related second additive compound matrix is assigned by,

$$J^{[2]} = \begin{pmatrix} -(M_{11}+M_{22}) & \sigma s & 0 & \sigma s & 0 & 0 \\ \rho & -(M_{11}+M_{33}) & 0 & 0 & 0 & 0 \\ \beta & \gamma & -(M_{11}+M_{44}) & 0 & 0 & \sigma s \\ 0 & \sigma i & 0 & -(M_{22}+M_{33}) & 0 & 0 \\ -\alpha & 0 & \sigma i & \gamma & -(M_{22}+M_{44}) & \sigma s \\ 0 & -\alpha & 0 & -\beta & \rho & -(M_{33}+M_{44}) \end{pmatrix}$$

Now, we assume the function:

$$P(s,e,i,q) = diag(1,1,e,e,1,1).$$
$$\Rightarrow P_f P^{-1} = diag\left(0,0,\frac{e'}{e},\frac{e'}{e},0,0\right)$$

Next, we find that,$PJ^{[2]}$

$PJ^{[2]}$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e & 0 & 0 & 0 \\ 0 & 0 & 0 & e & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} -(M_{11}+M_{22}) & \sigma s & 0 & \sigma s & 0 & 0 \\ \rho & -(M_{11}+M_{33}) & 0 & 0 & 0 & 0 \\ \beta & \gamma & -(M_{11}+M_{44}) & 0 & 0 & \sigma s \\ 0 & \sigma i & 0 & -(M_{22}+M_{33}) & 0 & 0 \\ -\alpha & 0 & \sigma i & \gamma & -(M_{22}+M_{44}) & \sigma s \\ 0 & -\alpha & 0 & -\beta & \rho & -(M_{33}+M_{44}) \end{pmatrix}$$

$$= \begin{pmatrix} -(M_{11}+M_{22}) & \sigma s & 0 & \sigma s & 0 & 0 \\ \rho & -(M_{11}+M_{33}) & 0 & 0 & 0 & 0 \\ \beta e & \gamma e & -(M_{11}+M_{44})e & 0 & 0 & \sigma s e \\ 0 & \sigma i e & 0 & -(M_{22}+M_{33})e & 0 & 0 \\ -\alpha & 0 & \sigma i & \gamma & -(M_{22}+M_{44}) & \sigma s \\ 0 & -\alpha & 0 & -\beta & \rho & -(M_{33}+M_{44}) \end{pmatrix}$$

Now since,

(4.9) $B = P_f P^{-1} + PJ^{[2]}P^{-1}$

$\Rightarrow$ B

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{e'}{e} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{e'}{e} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} -(M_{11}+M_{22}) & \sigma s & 0 & \frac{\sigma s}{e} & 0 & 0 \\ \rho & -(M_{11}+M_{33}) & 0 & 0 & 0 & 0 \\ \beta e & \gamma e & -(M_{11}+M_{44}) & 0 & 0 & \sigma s e \\ 0 & \sigma i e & 0 & -(M_{22}+M_{33}) & 0 & 0 \\ -\alpha & 0 & \frac{\sigma i}{e} & \frac{\gamma}{e} & -(M_{22}+M_{44}) & \sigma s \\ 0 & -\alpha & 0 & -\frac{\beta}{e} & \rho & -(M_{33}+M_{44}) \end{pmatrix}$$

$$= \begin{pmatrix} -(M_{11}+M_{22}) & \sigma s & 0 & \frac{\sigma s}{e} & 0 & 0 \\ \rho & -(M_{11}+M_{33}) & 0 & 0 & 0 & 0 \\ \beta e & \gamma e & -(M_{11}+M_{44})+\frac{e'}{e} & 0 & 0 & \sigma s e \\ 0 & \sigma i e & 0 & -(M_{22}+M_{33})+\frac{e'}{e} & 0 & 0 \\ -\alpha & 0 & \frac{\sigma i}{e} & \frac{\gamma}{e} & -(M_{22}+M_{44}) & \sigma s \\ 0 & -\alpha & 0 & -\frac{\beta}{e} & \rho & -(M_{33}+M_{44}) \end{pmatrix}$$

Now, let

$$B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix},$$

where,

$$B_{11} = (-(M_{11}+M_{22})), B_{12} = \begin{pmatrix} \sigma s & 0 & \frac{\sigma s}{e} & 0 & 0 \end{pmatrix}, B_{21} = \begin{pmatrix} \rho \\ \beta e \\ 0 \\ -\alpha \\ 0 \end{pmatrix},$$

$$B_{22} = \begin{pmatrix} -(M_{11}+M_{33}) & 0 & 0 & 0 & 0 \\ \gamma e & -(M_{11}+M_{44})+\frac{e'}{e} & 0 & 0 & \sigma s e \\ \sigma i e & 0 & -(M_{22}+M_{33})+\frac{e'}{e} & 0 & 0 \\ 0 & \frac{\sigma i}{e} & \frac{\gamma}{e} & -(M_{22}+M_{44}) & \sigma s \\ -\alpha & 0 & -\frac{\beta}{e} & \rho & -(M_{33}+M_{44}) \end{pmatrix}$$

Matrix B's Lozinskií measure is,

$$v(B) \leq Max(g_1, g_2)$$

Where,

$$g_1 = v(B_{11}) + ||B_{12}|| \text{and } g_2 = ||B_{21}|| + v(B_{22}).$$

By Simple calculation, we can easily obtain,

$$v(B_{11}) = -(M_{11}+M_{22}), ||B_{12}|| = Max\left\{\sigma s, \frac{\sigma s}{e}\right\}, ||B_{21}|| = Max\{\rho, \beta e, -\alpha\}, v(B_{22}).$$

$$\therefore\ g_1 = v(B_{11}) + \left||B_{12}|\right| = -(M_{11} + M_{22}) + max\{\sigma s, \frac{\sigma s}{e}\}$$

$$g_2 = \left||B_{21}|\right| + v(B_{22}) = max\{\rho, \beta e, -\alpha\} + v(B_{22}).$$

We define $v(B_{2\ 2})$, for this purpose the matrix $B_{2\ 2}$ is now split as:

$$B_{22} = C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix},$$

Where,

$$C_{11} = -(M_{11} + M_{33}), C_{12} = (0 \quad 0 \quad 0 \quad 0), C_{21} = \begin{pmatrix} \gamma e \\ \sigma i e \\ 0 \\ -\alpha \end{pmatrix},$$

$$C_{22} = \begin{pmatrix} -(M_{11} + M_{44}) + \frac{e'}{e} & 0 & 0 & \sigma s e \\ 0 & -(M_{22} + M_{33}) + \frac{e'}{e} & 0 & 0 \\ \frac{\sigma i}{e} & \frac{\gamma}{e} & -(M_{22} + M_{44}) & \sigma s \\ 0 & -\frac{\beta}{e} & \rho & -(M_{33} + M_{44}) \end{pmatrix}$$

We specify the matrix C's Lozinskií measure which is given by:

$$v(C) \leq Max(g_3, g_4),$$

Where,

$$g_3 = v(C_{11}) + \left||C_{12}|\right| \text{and } g_4 = \left||C_{21}|\right| + v(C_{22}).$$

$$\therefore v(C_{11}) = -(M_{11} + M_{33}),\ \left||C_{12}|\right| = 0,\ \left||C_{21}|\right| = Max\{\gamma e, \sigma i e, -\alpha\}, \text{ and } v(C_{22}).$$

$$g_3 = v(C_{11}) + \left||C_{12}|\right| = -(M_{11} + M_{33}) + 0 = -(M_{11} + M_{33}$$

$$g_4 = \left||C_{21}|\right| + v(C_{22}) = max\{\gamma e, \sigma i e, -\alpha\} + v(C_{22}).$$

We determine $v(C_{2\ 2})$, for this purpose the matrix $C_{2\ 2}$ is now divided as:

$$C_{22} = F = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix},$$

Where,

$$F_{11} = \left(-(M_{11} + M_{44}) + \frac{e'}{e}\right), F_{12} = (0 \quad 0 \quad \sigma s e), F_{21} = \begin{pmatrix} 0 \\ \frac{\sigma i}{e} \\ 0 \end{pmatrix},$$

$$F_{22} = \begin{pmatrix} -(M_{22} + M_{33}) + \frac{e^{'}}{e} & 0 & 0 \\ \frac{\gamma}{e} & -(M_{22} + M_{44}) & \sigma s \\ -\frac{\beta}{e} & \rho & -(M_{33} + M_{44}) \end{pmatrix}.$$

We specify the matrix F's Lozinskií measure which is given by:

$$v(F) \leq Max(g_5, g_6),$$

Where,

$$g_5 = v(F_{11}) + \left||F_{12}|\right| \text{and } g_6 = \left||F_{21}|\right| + v(F_{22}),$$

Where,

$$v(F_{11}) = -(M_{11} + M_{44}) + \frac{e^{'}}{e}, \left||F_{12}|\right| = \sigma s e, \left||F_{21}|\right| = \frac{\sigma i}{e}, \text{ and } v(F_{22}).$$

$$g_5 = v(F_{11}) + \left||F_{12}|\right| = -(M_{11} + M_{44}) + \frac{e^{'}}{e} + \sigma s, \text{ and } g_6 = \left||F_{21}|\right| + v(F_{22}) = \frac{\sigma i}{e} + v(F_{22}).$$

Now, we specify $v(F_{2\ 2})$, so $F_{2\ 2}$ is now separated as :

$$F_{22} = G = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix},$$

Where,

$$G_{11} = \left(-(M_{22} + M_{33}) + \frac{e^{'}}{e}\right), G_{12} = (0 \quad 0), G_{21} = \begin{pmatrix} \frac{\gamma}{e} \\ -\frac{\beta}{e} \end{pmatrix},$$

$$G_{22} = \begin{pmatrix} -(M_{22} + M_{44}) & \sigma s \\ \rho & -(M_{33} + M_{44}) \end{pmatrix}.$$

We specify the matrix G's Lozinskií measure which is given by:

$$v(G) \leq Max(g_7, g_8),$$

Where,

$$g_7 = v(G_{11}) + \left||G_{12}|\right| \text{and } g_8 = \left||G_{21}|\right| + v(G_{22}),$$

Where,

$$v(G_{11}) = -(M_{22} + M_{33}) + \frac{e^{'}}{e}, \left||G_{12}|\right| = 0, \left||G_{21}|\right| = Max\left\{\frac{\gamma}{e}, -\frac{\beta}{e}\right\},$$

$$v(G_{22}) = \text{Max}\{-(M_{22} + M_{44}) + \sigma s, \rho - (M_{33} + M_{44})\}.$$

$$g_7 = v(G_{11}) + ||G_{12}|| = -(M_{22} + M_{33}) + \frac{e'}{e} + 0 = -(M_{22} + M_{33}) + \frac{e'}{e}$$

and $g_8 = ||G_{21}|| + v(G_{22}) = Max\left(\frac{\gamma}{e}, -\frac{\beta}{e}\right) + Max\{-(M_{22} + M_{44}) + \sigma s, \rho - (M_{33} + M_{44})\}$

Therefore,

$$v(G) \leq Max(g_7, g_8)$$

$$\leq Max\ [-(M_{22} + M_{33}) + \frac{e'}{e}, \{Sup\left(\frac{\gamma}{e}, -\frac{\beta}{e}\right) + Sup\{-(M_{22} + M_{44})\& + \sigma s, \rho - (M_{33} + M_{44})\}]$$

$$\Rightarrow v(G) \leq \frac{e'}{e} + M$$

Now,

$$v(F) \leq Max(g_5, g_6) \leq Max\{-(M_{11} + M_{44}) + \frac{e'}{e} + \sigma se, \frac{\sigma i}{e} + v(F_{22}).$$

$$\Rightarrow v(F) \leq \frac{e'}{e} + Max\ [-(M_{11} + M_{44}) + \sigma se, \frac{\sigma i}{e} - (M_{22} + M_{33}), \frac{\sigma i}{e} + \sigma s - \frac{e'}{e} - (M_{22} + M_{44}) + Sup\left(\frac{\gamma}{e}, -\frac{\beta}{e}\right), \frac{\sigma i}{e} + \rho - (M_{33} + M_{44}) + Sup\ \left(\frac{\gamma}{e}, -\frac{\beta}{e}\right)]$$

Also,

$$v(C) \leq Max\ (g_3, g_4) \leq Max\ \{(M_{11} + M_{33}), Sup\ (\gamma e, \sigma ie, -\alpha) + v(C_{22})\}.$$

$$\Rightarrow v(C) \leq \frac{e'}{e} + Max\{(M_{11} + M_{33}) - \frac{e'}{e}, -(M_{11} + M_{44}) + \sigma se + Sup\ (\gamma e, \sigma ie, -\alpha), \frac{\sigma i}{e} - (M_{22} + M_{33}) + Sup(\gamma e, \sigma ie, -\alpha), \frac{\sigma i}{e} + \sigma s - \frac{e'}{e} - (M_{22} + M_{44}) + Sup\ \left(\frac{\gamma}{e}, -\frac{\beta}{e}\right) + Sup(\gamma e, \sigma ie, -\alpha), \frac{\sigma i}{e} + \rho - \frac{e'}{e} - (M_{33} + M_{44}) + Sup\left(\frac{\gamma}{e}, -\frac{\beta}{e}\right) + Sup(\gamma e, \sigma ie, -\alpha)\}$$

and

$(4.10) v(B) \leq Max(g_1, g_2),$

$$\leq Max\ [-(M_{11} + +M_{22}) + Sup\left(\sigma s, \frac{\sigma s}{e}\right) + \frac{e'}{e} - \frac{e'}{e}, Sup\ (\rho, \beta e, -\alpha) + v(B_{22})]$$

$$\Rightarrow \upsilon(B) \leq \frac{e^{'}}{e} + Max\left[-(M_{11}+M_{22}) + Sup\left(\sigma s, \frac{\sigma s}{e}\right) - \frac{e^{'}}{e}, Sup(\rho, \beta e, -\alpha) + (M_{11}+M_{33}) - \frac{e^{'}}{e}, -(M_{11}+M_{44}) + \sigma s e + Sup(\gamma e, \sigma i e, -\alpha) + Sup(\rho, \beta e, -\alpha), \frac{\sigma i}{e} - (M_{22}+M_{33}) + Sup\,(\gamma e, \sigma i e, -\alpha) + Sup(\rho, \beta e, -\alpha), \frac{\sigma i}{e} + \sigma s - \frac{e^{'}}{e} - (M_{22}+M_{44})\right) + Sup\left(\frac{\gamma}{e}, -\frac{\beta}{e}\right) + Sup(\gamma e, \sigma i e, -\alpha) + Sup\,(\rho, \beta e, -\alpha), \frac{\sigma i}{e} + \rho - \frac{e^{'}}{e} - (M_{33}+M_{44}) + Sup\left(\frac{\gamma}{e}, -\frac{\beta}{e}\right) + Sup\,(\gamma e, \sigma i e, -\alpha) + Sup(\rho, \beta e, -\alpha)]$$

$$\Rightarrow \upsilon(B) \leq \frac{e^{'}}{e} - Min\,[(M_{11}+M_{22}) \pm Inf\,\left(-\sigma s, -\frac{\sigma s}{e}\right) + \frac{e^{'}}{e}, -Inf(-\rho, -\beta e, \alpha) - (M_{11}+M_{33}) + \frac{e^{'}}{e}, (M_{11}+M_{44}) - \sigma s e - Inf\,(-\gamma e, -\sigma i e, , \alpha) - Inf(-\rho, -\beta e, \alpha), -\frac{\sigma i}{e} + (M_{22}+M_{33}) - Inf\,(-\gamma e, -\sigma i e, \alpha) - Inf\,(-\rho, -\beta e, \alpha), -\frac{\sigma i}{e} - \sigma s + \frac{e^{'}}{e} + (M_{22}+M_{44}) - Inf\left(-\frac{\gamma}{e}, \frac{\beta}{e}\right) - Inf\,(-\gamma e, -\sigma i e, \alpha) - Inf\,(-\rho, -\beta e, \alpha), -\frac{\sigma i}{e} - \rho + \frac{e^{'}}{e} + (M_{33}+M_{44}) - Inf\,\left(-\frac{\gamma}{e}, \frac{\beta}{e}\right) - Inf\,(-\gamma e, -\sigma i e, \alpha) - Inf\,(-\rho, -\beta e, \alpha)]$$

$$\Rightarrow \upsilon(B) \leq \frac{e^{'}}{e} - \bar{b}\,,$$

Where,

$$(4.11)\bar{b} = Min\,[(M_{11}+M_{22}) \pm Inf\,\left(-\sigma s, -\frac{\sigma s}{e}\right) + \frac{e^{'}}{e}, -Inf(-\rho, -\beta e, \alpha) - (M_{11}+M_{33}) + \frac{e^{'}}{e}, (M_{11}+M_{44}) - \sigma s e - Inf\,(-\gamma e, -\sigma i e, , \alpha) - Inf(-\rho, -\beta e, \alpha), -\frac{\sigma i}{e} + (M_{22}+M_{33}) - Inf\,(-\gamma e, -\sigma i e, \alpha) - Inf\,(-\rho, -\beta e, \alpha), -\frac{\sigma i}{e} - \sigma s + \frac{e^{'}}{e} + (M_{22}+M_{44}) - Inf\left(-\frac{\gamma}{e}, \frac{\beta}{e}\right) - Inf\,(-\gamma e, -\sigma i e, \alpha) - Inf\,(-\rho, -\beta e, \alpha), -\frac{\sigma i}{e} - \rho + \frac{e^{'}}{e} + (M_{33}+M_{44}) - Inf\,\left(-\frac{\gamma}{e}, \frac{\beta}{e}\right) - Inf\,(-\gamma e, -\sigma i e, \alpha) - Inf\,(-\rho, -\beta e, \alpha)]\,.$$

Consider a solution $((s(t), e(t), i(t), q(t), r(t))$ arising from the absorbing compact set $\Gamma \subset \Omega$. Let $\bar{t}$ be sufficient large such that the system is persistence and $\left(s(t), e(t), i(t), q(t), r(t)\right) \subset \Gamma$, $\forall\, t > \bar{t}$.

Then for all outcome $((s(t), e(t), i(t), q(t), r(t)) \in \Omega$ with $(s(0), e(0), i(0), q(0), r(0)) \in \Gamma$, for $t > \bar{t}$,

$$\frac{1}{t}[ln\, e(t) - ln\, e(0)] < \frac{\bar{b}}{2}\,.$$

Finally we get,

$$\frac{1}{t}\int_0^t v(B)ds \;\; \leq \frac{1}{t}\int_0^t \left(\frac{e^{'}}{e} - \bar{b}\right) ds = \frac{1}{t}\,[ln\, e(t) - ln\, e(0) - \bar{b}(t)]$$

$$= \frac{[ln\, e(t) - ln\, e(0)]}{t} - \bar{b} \leq \; -\frac{\bar{b}}{2}.$$

$$(4.12) \;\Rightarrow\; \bar{q}_2 \leq \; -\frac{\bar{b}}{2} < 0.$$

Hence, $x_{EE} \in \; Int\; \Omega$ is asymptotically stable globally, when $R_0 > 1\; and\; \bar{b} > 0$.

This finalizes the theorem.

5. Sensitivity Analysis

As the basic reproduction number is threshold quantity to determine the prediction of infection. The sensitivity analysis indicate that which variables is produce more effect on the value of basic reproduction number. The formula for sensitivity of R_0 with respected to variable p is given by-

$$\kappa_V^{R_0} = \frac{v}{R_0} * \frac{\partial R_0}{\partial v}$$

Positive and negative index indicate that proportional and inversely proportional relation of variable with R_0 . We calculate the sensitivity index of one variable by taking 0.001 values of all other parameters. We easily observe that increasing values of $\alpha,\; \beta\; and\; \gamma$ can cause to reduce the value of R_0, on the other hand , the increasing value of $\lambda\,, \rho\; and\; \sigma$ can increase the value of R_0.

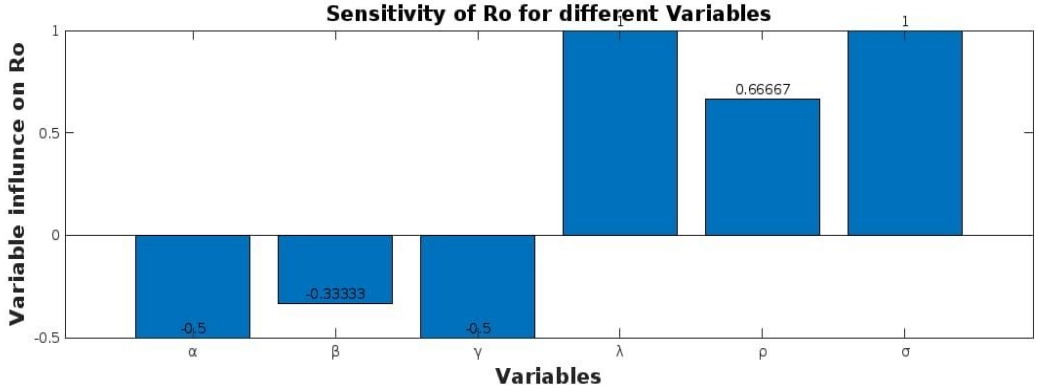


Figure 5.1 Sensitivity of variables with basic reproduction number.

6. Simulations and Effect of Quarantine Strategies

Now, we take various values of variables to test the validity of model by applying simulation using *MATLAB* software. Let S=25000,E=I=Q=R=100 . See ***Table 5.1*** for the values of other variables.

Table 5.1 Value of variables.

Variables	**Case 1**	**Case 2**
α	0.0001	0.025
β	0.07	0.07
γ	0.09	0.09
λ	0.028	0.028
δ	0.0001	0.1
Φ	0.0001	1
μ	0.017	0.017
σ	0.0001	0.3
ρ	0.0001	0.7
R_0	<1	1.66 (>1)

Taking S = 25000, E=10000, I= 20000, Q=10000, and R= 10000 i.e. s=0.3333, e=q=r=1.333, i=0.2667 and using case 2 values we obtain eigen values from section 4.3 :

$\lambda = -0.117, -1.017, -0.9073, -0.0259 + 0.1025i, -0.0259 - 0.1025i$ that all are negative real parts hence x_{EE} is locally asymptotically stable confirmed numerically.

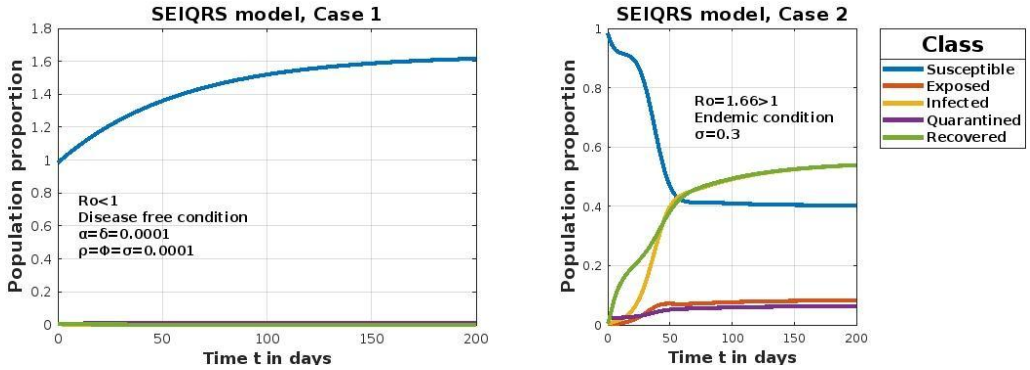
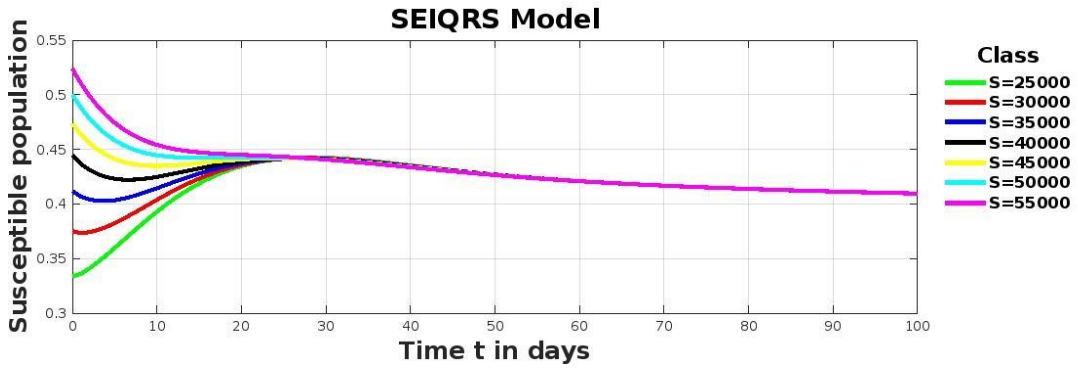


Figure 5.2: In case 1, the epidemic curve exhibits the disease free behavior and proportion of susceptible increases, since in this case $R_0 < 1$.

In case 2, the epidemic curve exhibits the disease endemic behavior. Proportion of susceptible decreases, proportion of exposed and infected increases, since in this case $R_0 > 1$.



.

Figure 5.3in case 2, $R_0 > 1$, the fraction of susceptible decreases continuously , and approaches to 0.4010. As one infected person spreads the disease, and infect more than one susceptible. Susceptible population reduced to a saturated level. Hence endemic equilibrium point is locally as well as globally asymptotically stable.

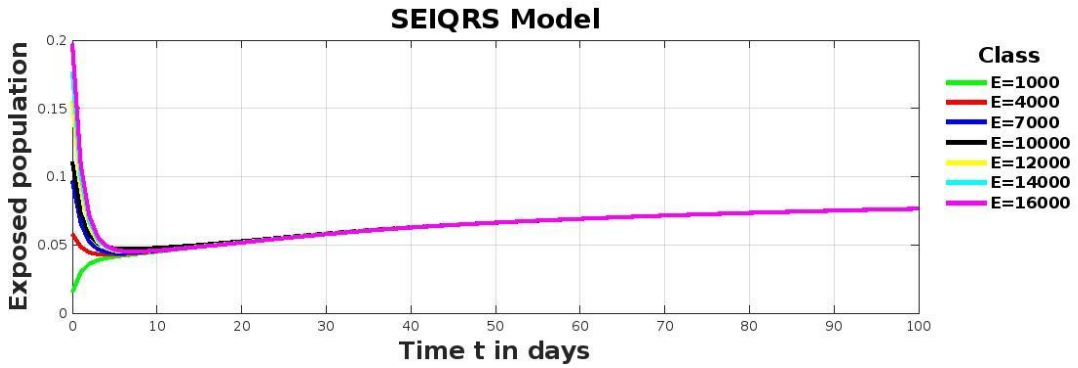


Figure 5.4 In case 2, $R_0 > 1$, the fraction of exposed decreases initially (It may be due to unawareness to the epidemic , and after that it increased rapidly (Due to more testing and awareness) and at last it approaches to 0.0839.

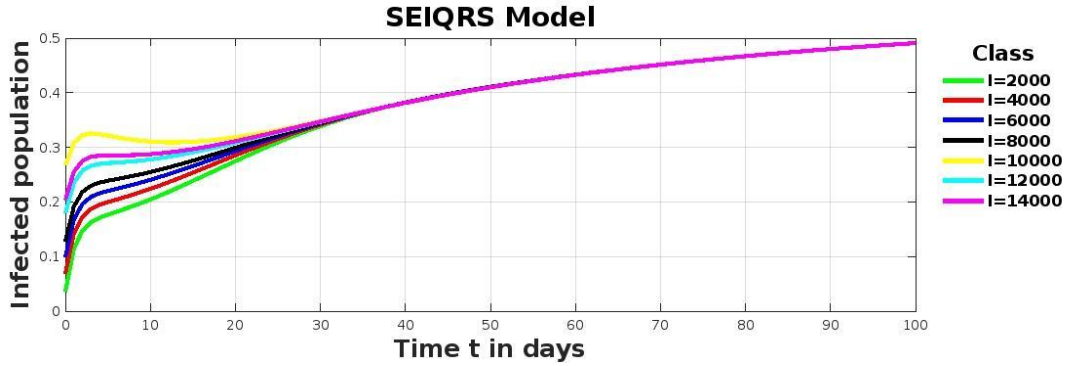


Figure 5.5 In case 2, $R_0 > 1$, the fraction of infected individuals increased continuously Since one infected person infect more than one susceptible, and positive testing of exposed individuals confirmed the status of infected, that result the increment in the infected class. and at last it approaches to 0.5490.

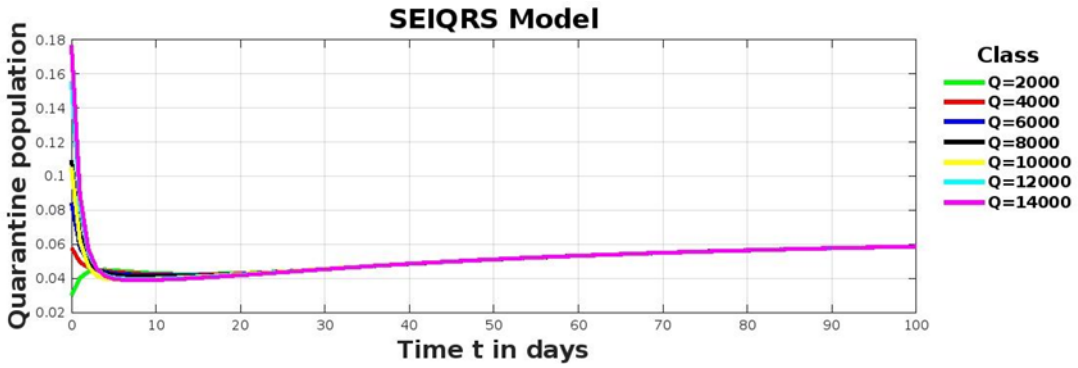


Figure 5.6In case 2, $R_0 > 1$, the fraction of quarantined individuals decreased initially (It may be due to not following guidelines of government and people did not take the epidemic seriously) , but after the increments in infected the fraction of quarantined individuals increased, and after some time it tends to 0.0642 (It may be due to fear and more awareness regarding the epidemic).

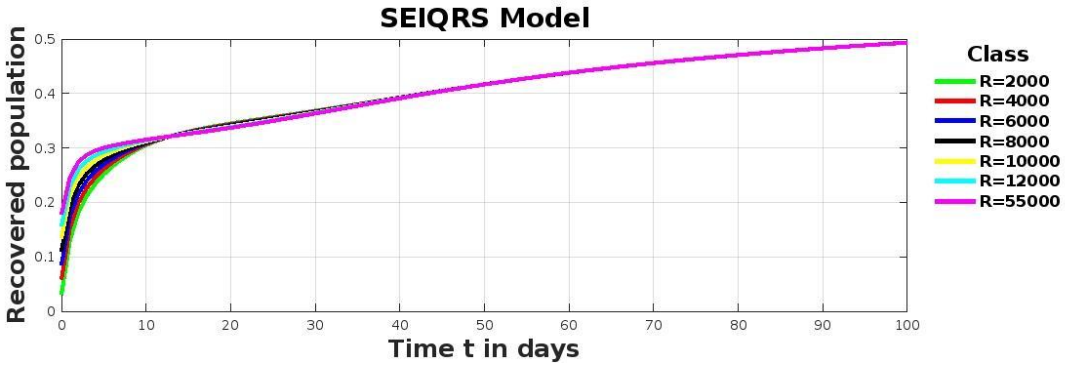


Figure 5.7 In case 2, $R_0 > 1$, fraction of recovered individuals increased in this case also, because we assume that each and every person, either recovered from disease or death due to disease enter into recovered class. After some time, the fraction of recovered individuals approaches to 0.5490.

Thus, according to the *Theorem* 4.5, when,$R_0 > 1$, the point x_{EE} is globally asymptotically stable.

Further, effect of quarantine/isolation strategies depicted in the following figures. We take the case 2 values of parameters of table 5.1 and change the rates $\alpha,\ \beta\ and\ \gamma$ to see how it effects to control the COVID-19.

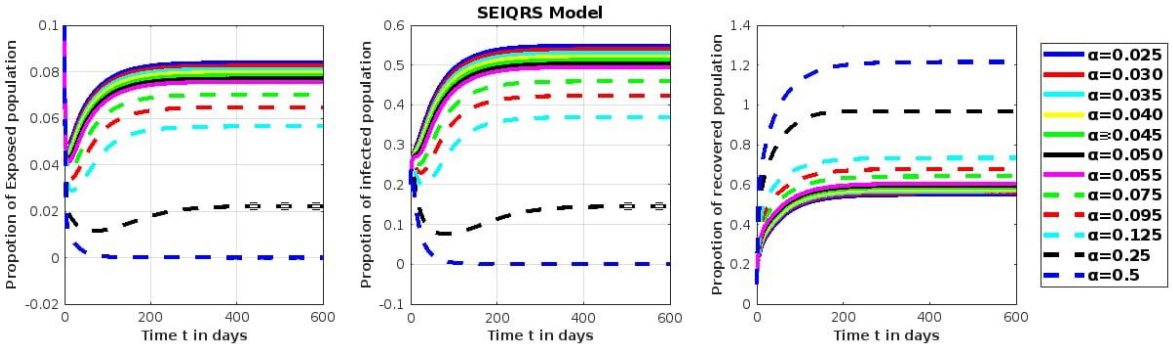


Figure 5.8 In this figure, as the value of quarantine/isolation rate αincreases the exposed population, infected population and the value of R_0 decreases and the recovered increases as comparatively.

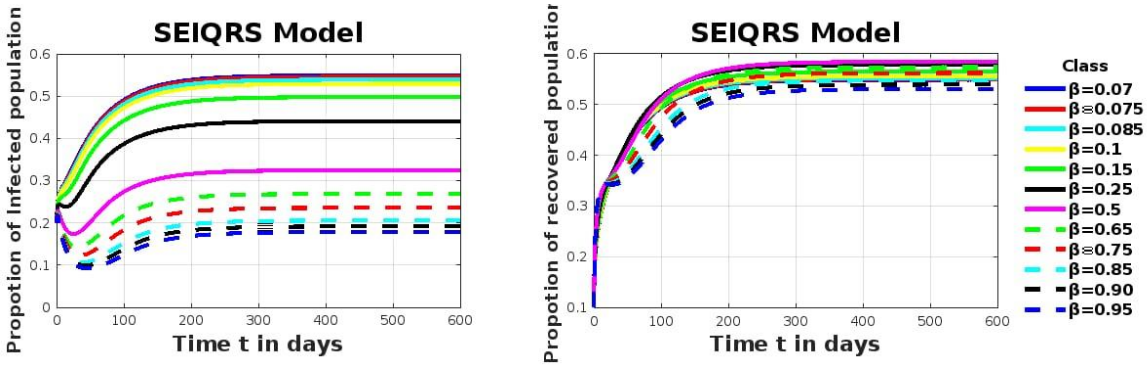


Figure 5.9 In this figure, as the value of quarantine/isolation rate βincreases the infected population and the value of R_0 decreases and the recovered increases as comparatively.

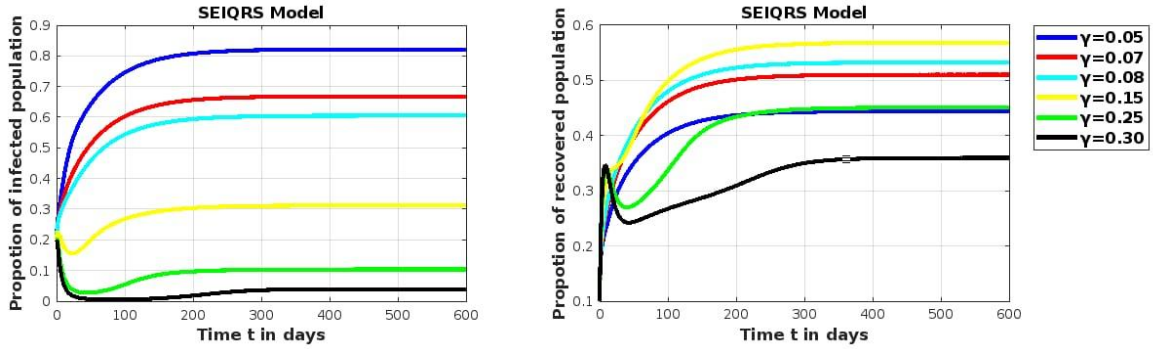


Figure 5.10 In this figure, as the value of isolation rateγincreases the infected population and the value of R_0 decreases as comparatively and the recovered increased till $\gamma = 0.15$ but further values decline the recovered graph due to declination of infected population.

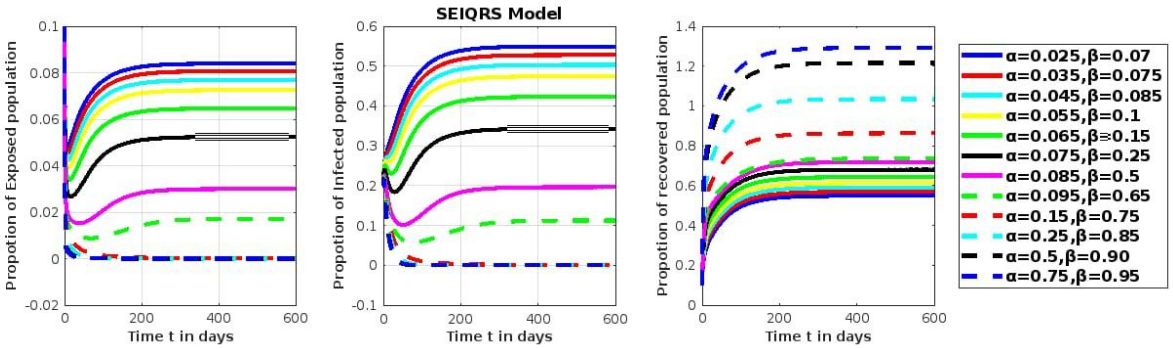


Figure 5.11 In this figure, as the values ofα and βincreases simultaneously the infected population and the value of R_0 decreases and the recovered increases as comparatively

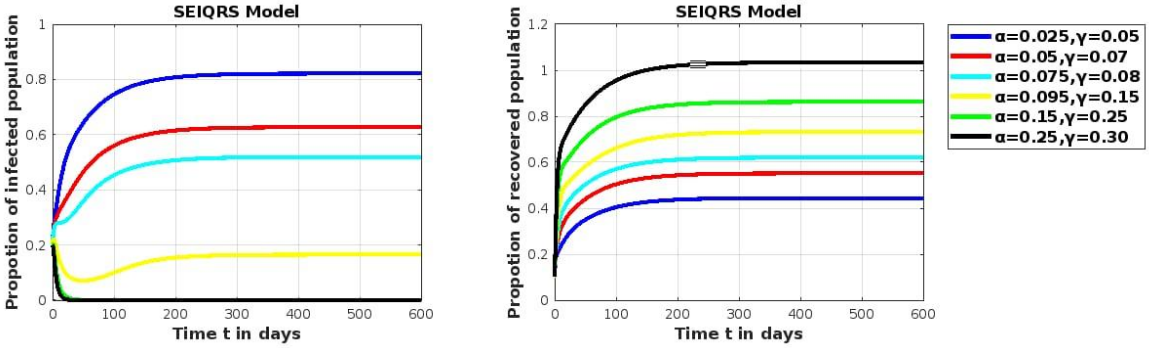


Figure 5.12 In this figure, as the values ofα and γincreases simultaneously the infected population and the value of R_0 decreases and the recovered population increases as comparatively.

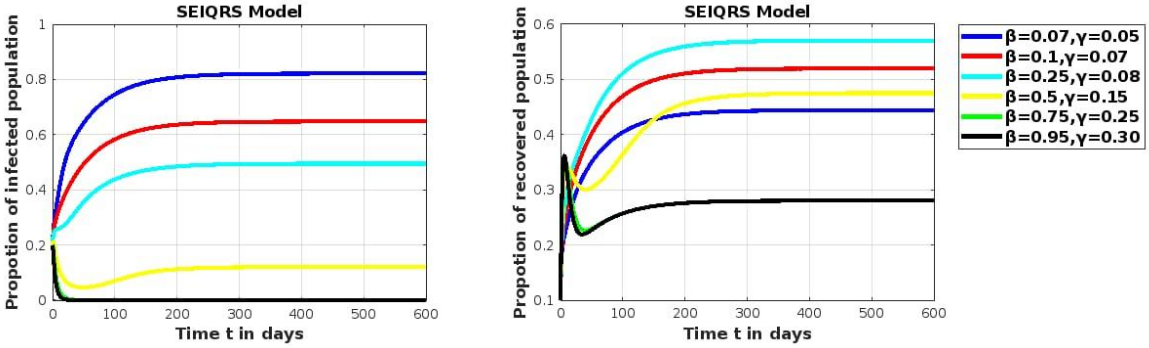


Figure 5.13 In this figure, as the values β and γincreases simultaneously the infected population and the value of R_0 decreases and the recovered population increases as comparatively.

Obviously, increment of quarantine/isolation rate can reduce the exposed and infected population. It significantly affected the basic reproduction number. Therefore, it is important to imply the quarantine/isolation strategies effectively to overcome the disease.Moreover, simultaneously increasing of isolation/quarantine rates can reduce the infection more effectively, as compare to separately increasing of that rate.

6.Conclusions

Our result exhibits that if the quarantine/isolation strategies imposed effectively, it can definitely reduce the chance of spreading of disease. Quarantine of susceptible population and isolation of exposed and infected population can deduct the number of infected people effectively. These prevention strategies can decrease the basic reproduction number. Our study depicts the importance of quarantine/isolation strategies to stop the rapid spreading of an infectious disease.

8. Conflict of Interest

We declare that there is no conflict of interest.

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