Challenges in Classical and Non-Newtonian Fluid Mechanics A Mathematical Perspective

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Abstract

Challenges in classical and non-Newtonian fluid mechanics are abundant, particularly when viewed from a mathematical perspective. Classical fluid mechanics, based on Newtonian principles, deals with the behavior of simple fluids like water and air. However, challenges arise when dealing with complex fluids that do not follow Newton's linear relationship between stress and strain rate.One significant challenge is the mathematical description of non-Newtonian fluids, which often require sophisticated constitutive equations to capture their behavior accurately. These equations must account for shear-thinning, shear-thickening, viscoelasticity, and other complex rheological properties. Finding appropriate mathematical models that can predict these behaviors under various conditions is a formidable task.Additionally, solving the governing equations of fluid mechanics, the Navier-Stokes equations, poses significant mathematical challenges, especially in the context of non-Newtonian fluids. These equations are notoriously difficult to solve analytically and often require numerical methods, which can be computationally intensive. Understanding the stability and turbulence of non-Newtonian flows is another mathematical challenge. Turbulent flows of complex fluids exhibit unique characteristics that differ from those of Newtonian fluids, requiring advanced mathematical tools to analyze and predict their behaviour.

Keywords:-Viscosity,ClassicalNewtonian fluid,Non-Newtonian Fluid,

Introduction

Challenges in classical and non-Newtonian fluid mechanics are at the forefront of scientific inquiry and engineering applications, offering a captivating field for exploration and innovation.

These challenges emerge from the intricate interplay of physical phenomena and mathematical descriptions that govern the behavior of fluids, ranging from everyday substances like water and air to complex materials with unique rheological properties.Classical fluid mechanics, grounded in Sir Isaac Newton's pioneering work, provides a fundamental framework for understanding the dynamics of simple, Newtonian fluids. However, even in this well-established realm, challenges persist. For instance, the accurate prediction of fluid flow in practical engineering scenarios remains a formidable task. Turbulence, a chaotic and highly complex state of fluid motion, continues to elude a complete mathematical description, making it one of the central enigmas in classical fluid mechanics.

When transitioning to non-Newtonian fluids, which encompass a wide range of materials such as polymers, slurries, and biological fluids, the complexity escalates. These substances exhibit behaviors that deviate significantly from the linear relationship between stress and strain rate characteristic of Newtonian fluids. Challenges arise in the formulation of mathematical models that can capture the diverse rheological properties of non-Newtonian fluids, including shear-thinning, shear-thickening, and viscoelasticity.

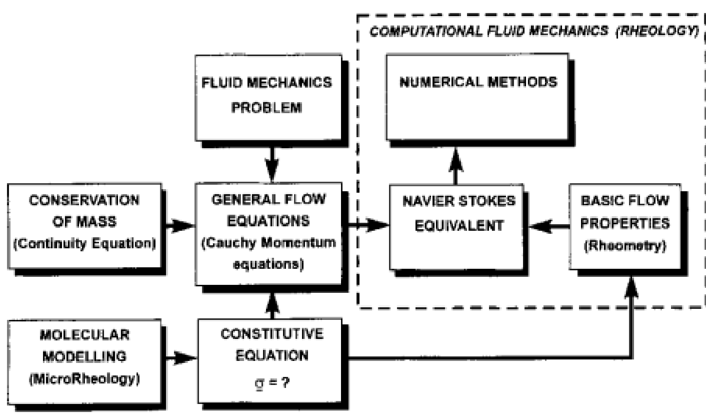

Fig 1 Non-Newtonian fluids mechanics problem

A fundamental challenge lies in the development of constitutive equations that accurately represent the complex stress-strain relationships exhibited by non-Newtonian fluids. These equations must account for the effects of varying shear rates, temperature, and pressure, making them highly intricate and dependent on the specific fluid in question. Consequently, understanding and predicting the flow behavior of non-Newtonian fluids under different conditions necessitate a deep understanding of these mathematical models. The solution of the governing equations of fluid mechanics, the Navier-Stokes equations, poses a substantial mathematical challenge, particularly for non-Newtonian flows. Analytical solutions are often elusive, necessitating the utilization of numerical methods that can be computationally intensive and require advanced computational resources.

The usual scaling

Usual set of choices of characteristic quantities

In the process of non-dimensionalizing a fluid mechanics problem, the initial step typically involves selecting a characteristic velocity. In the context of internal flows, a common and representative choice is to use the average velocity at a specific cross-sectional area, denoted as A.

$$v_c = \frac{1}{A}\int_A |\boldsymbol{v}\cdot\boldsymbol{n}|\,\mathrm{d}A$$

Alternatively, a characteristic viscosity can be defined by combining characteristic values of the pressure gradient, viscosity, and length. This provides another approach to non-dimensionalizing a fluid mechanics problem, in addition to using characteristic velocity as mentioned earlier.

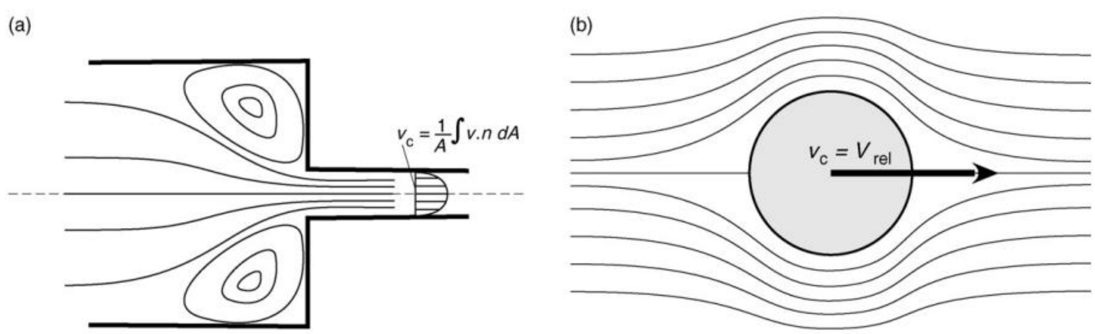

Fig. 2. Typical choices for characteristic velocity for (a) internal flows and (b) external flows.

For external flows, the far-away fluid velocity relative to thesolid boundary, Vrel, is the usual selected characteristic velocity:

$$v_c = V_{rel}$$

In the context of analyzing flows involving non-Newtonian fluids, it's essential to consider that the stress-response relationship is nonlinearly dependent on flow kinematics. Therefore, to characterize and analyze such flows effectively, it becomes crucial to introduce a characteristic deformation rate, denoted as γ˙c. Typically, this characteristic deformation rate is defined as the ratio of a previously chosen characteristic velocity to a characteristic length, denoted as L.

$$\dot{\gamma}_c = \frac{v_c}{L}$$

In scenarios where the flow is primarily characterized as a shear flow, the characteristic length L should be a measurement taken perpendicular to the direction of the flow. Conversely, in situations where the flow is predominantly extensional, it is advisable for L to represent a length measured along the primary flow direction.

Indeed, when selecting the characteristic deformation rate for a given analysis, it is essential to ensure that it falls within the range of deformation rates actually observed in the flow. Furthermore, the rheological properties and information used in the analysis must correspond to the same range of deformation rates to maintain consistency and relevance in the study.

In cases involving periodic flows, it is often practical to designate the inverse of a characteristic frequency as the characteristic time. However, for other types of flows, a characteristic time can be defined as the reciprocal of the characteristic deformation rate, denoted as tc = 1/γ˙c = L/vc. In essence, this characteristic time represents the duration it takes for a fluid particle traveling at the speed vc to cover a distance equal to the characteristic length L, as per its definition.

Characteristic values of other significant flow parameters can be derived from the previously selected characteristic quantities. Specifically, the characteristic viscosity is determined by evaluating the viscosity function η(γ˙) at the characteristic deformation rate, which is defined as part of the non-dimensionalization process. This allows for the establishment of a comprehensive set of characteristic values that describe the flow behavior effectively.

$$\eta_c \equiv \eta \left(\frac{v_c}{L} \right)$$

In a similar manner, a characteristic stress can be established by multiplying the characteristic viscosity by the characteristic deformation rate. Mathematically, this is expressed as τc = ηcvc/L. Additionally, other characteristic quantities that are required for the analysis can also be defined using analogous relationships, ensuring a consistent and comprehensive characterization of the flow under consideration.

Parameters in the momentum equation

For incompressible materials, the mass conservation equation is written as

$$\boldsymbol{\nabla}.\boldsymbol{v} = \mathbf{0}$$

where v is the velocity vector field

The linear momentum conservation principle requires that

$$\rho \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho \boldsymbol{g}$$

where ρ is the mass density, d(v)/dt the material time derivative, p the pressure, τ ≡ T + p1 the extra-stress tensor field, T the stress tensor field, 1 the unit tensor, and g is a field of external forces (e.g. gravity)

The subsequent step in the process of non-dimensionalization involves utilizing the chosen characteristic quantities to derive dimensionless versions of the conservation equations. To achieve this, the following dimensionless variables are defined:

$$t^* \equiv \frac{v_c}{L} t; \qquad \boldsymbol{v}^* \equiv \frac{\boldsymbol{v}}{v_c}; \qquad \nabla^* \equiv L\nabla; \qquad p^* \equiv \frac{pL}{\eta_c v_c};$$

$$\boldsymbol{\tau}^* \equiv \frac{\boldsymbol{\tau} L}{\eta_c v_c}$$

The dimensionless version of the conservation equations is

$$\nabla^* \cdot \boldsymbol{v}^* = 0$$

$$Re \frac{\mathrm{d}\boldsymbol{v}^*}{\mathrm{d}t^*} = -\nabla^* p^* + \nabla^* \cdot \boldsymbol{\tau}^* + Ga \frac{\boldsymbol{g}}{g}$$

When dealing with fluid dynamics, specifically in the study of fluid flow, two dimensionless groups become relevant: the Reynolds number (Re) and the Galilei number (Ga). The Reynolds number serves as an indicator of the significance of inertia forces compared to viscous forces within the flow.

$$Re \equiv \frac{\rho v_c L}{\eta_c}$$

The Galilei number gives the importance of the external forcesrelative to viscous forces in the flow.

$$Ga \equiv \frac{\rho g L^2}{\eta_c v_c}$$

In situations involving flows with free or moving boundaries, additional dimensionless groups may become relevant, such as the capillary number. The capillary number, often denoted as Ca, is a dimensionless parameter that characterizes the balance between viscous forces and capillary (surface tension) forces in a fluid flow. It can be expressed as follows:

$$Ca \equiv \frac{\eta_c v_c}{\sigma}$$

Where σ is the surface tension.

The provided definitions are offered solely as an illustrative example of the non-dimensionalization procedure. It's evident that these definitions are not the only options available, and different characteristic quantities and dimensionless parameters may be used depending on the specific nature of the problem at hand. The appropriate choices are made to ensure that the dimensionless variables and their corresponding derivatives, which appear in the conservation equations, have magnitudes of the order of one (O(1)). Consequently, the relative magnitudes of the terms within these equations align with those of the dimensionless groups that appear as coefficients.

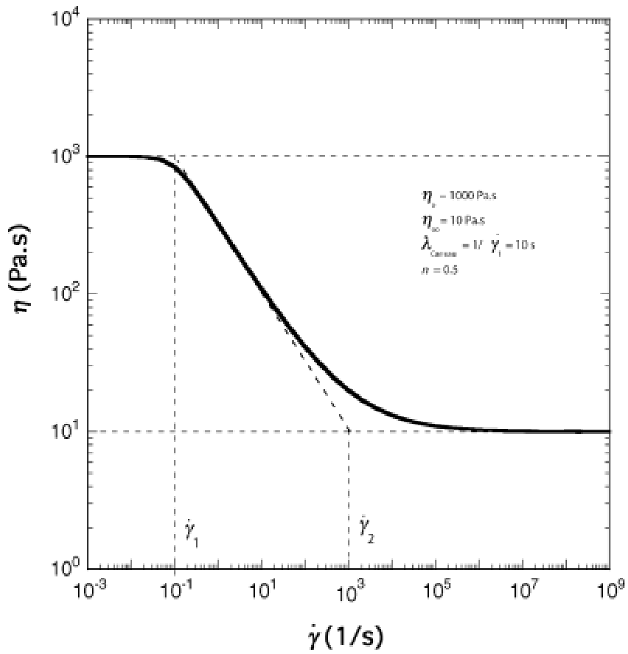

Fig. 3. The Carreau viscosity function

Viscoelastic liquids

For Boger liquids, η∗ = 1, and thus there is no transition shear rate to be used as characteristic deformation rate. In this case.

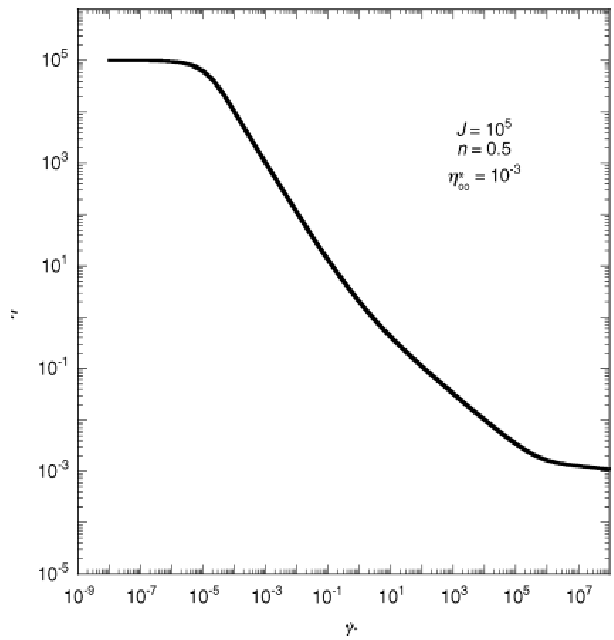

Fig. 4 The dimensionless viscosity function for viscoplastic liquids.

Importance of the research

Research into the challenges of classical and non-Newtonian fluid mechanics from a mathematical perspective is of paramount importance. These studies form the cornerstone of our understanding of fluid behavior, impacting fields as diverse as aerospace, environmental science, healthcare, and materials science. In aerospace, precise mathematical models are essential for designing efficient aircraft and rockets. In healthcare, understanding blood flow and drug delivery relies on fluid mechanics, while innovations in medical devices depend on mathematical insights. The environmental sector benefits from predicting and mitigating natural disasters and pollution transport. Industries from pharmaceuticals to petrochemicals optimize processes through mathematical models, reducing waste and enhancing product quality. Space exploration and energy efficiency also rely on fluid dynamics research, as do advancements in materials science. In essence, the mathematical exploration of fluid mechanics is the bedrock upon which countless scientific and technological advancements are built, offering solutions to real-world challenges and driving progress across multiple domains.

Literature Review

Zvyagin, V. G., &Orlov, V. P. (2018). The solvability of non-Newtonian fluid dynamics models incorporating memory effects presents a significant challenge in the field of mathematical fluid mechanics. These models, often characterized by integro-differential equations, arise in various practical applications, including the study of complex fluids, polymers, and biological systems. This abstract focuses on the investigation of the solvability of such non-Newtonian fluid models with memory, emphasizing the mathematical aspects.The primary objective of this research is to establish the existence and uniqueness of solutions for these models, which are crucial for accurate predictions and engineering applications. To achieve this, we delve into the intricate mathematical framework involving fractional derivatives, convolution integrals, and rheological constitutive equations.

Crochet, M. J.,(2012) Numerical simulation of non-Newtonian flow is indispensable for understanding and harnessing the intricate behavior of these complex fluids. In industries spanning from food processing to pharmaceuticals and from oil drilling to biomedicine, the ability to model and predict the flow of non-Newtonian fluids is pivotal. Such fluids exhibit diverse rheological characteristics, making them particularly challenging to describe mathematically and experimentally. Numerical simulations serve as a cost-effective and efficient means to explore their behavior under various conditions and geometries, offering engineers and researchers valuable insights into system design, process optimization, and troubleshooting. Moreover, these simulations aid in the development and validation of constitutive models, enabling accurate representations of non-Newtonian fluid behavior in a wide array of applications. Ultimately, numerical simulations empower innovation and problem-solving across multiple industries, making them an invaluable tool in the study of non-Newtonian flow.

Anand, V., David Jr, J., &Christov, I. C. (2019). The study of Non-Newtonian fluid–structure interactions, specifically the static response of a microchannel induced by the internal flow of a power-law fluid, is a significant and challenging area of research in fluid mechanics and microfluidics. This research investigates the behavior of non-Newtonian fluids, which deviate from the classical linear relationship between shear stress and shear rate observed in Newtonian fluids.In microfluidic devices, where the characteristic length scales are small, the influence of fluid properties on the structural integrity of the channels becomes pronounced. Power-law fluids, characterized by a viscosity that varies with the power of shear rate, exhibit a range of complex behaviors, including shear-thinning (decreasing viscosity with increasing shear rate) or shear-thickening (increasing viscosity with shear rate).Researchers employ computational fluid dynamics (CFD) simulations, analytical models, and experimental techniques to analyze the static response, flow-induced deformation, and stress distribution within microchannels. This investigation aids in predicting potential problems, such as channel collapse or blockage, and guides the development of more efficient microfluidic devices for applications in healthcare, chemical analysis, and materials science.

Chhabra, R. P. (2010). Non-Newtonian fluids, with their intriguing and diverse rheological properties, represent a captivating area of study in the realm of fluid mechanics and material science. These fluids defy the simplicity of Newtonian behavior, where viscosity remains constant regardless of shear rate, and instead, they exhibit a wide spectrum of responses to external forces. From shear-thinning fluids like ketchup that become more fluid when agitated, to shear-thickening mixtures like cornstarch and water that solidify upon impact, non-Newtonian fluids challenge our conventional understanding of fluid dynamics. Moreover, the viscoelasticity observed in substances such as polymers and biological tissues introduces a fascinating blend of solid-like and liquid-like behavior, making them invaluable in various industries and scientific disciplines. The rheology of non-Newtonian fluids, described through mathematical models and constitutive equations, underpins their application in fields ranging from engineering and pharmaceuticals to food processing and biology.

Research Problem

Research in classical and non-Newtonian fluid mechanics presents a rich landscape of mathematical challenges that extend beyond the confines of conventional fluid dynamics. While classical fluid mechanics, governed by the Navier-Stokes equations, has provided a solid foundation for understanding the behaviour of Newtonian fluids, the complexities introduced by non-Newtonian fluids demand rigorous mathematical examination. These fluids, characterized by intricate relationships between stress and strain, are pervasive in numerous industries, from polymer processing to food science and biomedicine. Addressing these challenges requires a multifaceted approach that explores mathematical modeling, the existence and regularity of solutions, and the impact of non-Newtonian properties on flow dynamics.One pressing issue lies in developing mathematical models that can accurately represent the diverse behaviors of non-Newtonian fluids under varying conditions. This encompasses understanding viscoelasticity, shear-thinning, and yield-stress behaviour. Moreover, researchers need to delve into the existence and regularity of solutions to the Navier-Stokes equations for non-Newtonian fluids, ensuring the stability and convergence of numerical simulations. Investigating how non-Newtonian properties influence flow phenomena like turbulence, boundary layers, and separation compared to classical fluids is another critical avenue.

Conclusion

The field of classical and non-Newtonian fluid mechanics is a fascinating and complex area of study that presents a multitude of challenges. One of the primary challenges in classical fluid mechanics is the development of accurate mathematical models to describe the behavior of fluids under various conditions. The Navier-Stokes equations, which govern the motion of Newtonian fluids, remain unsolved for many practical situations, such as turbulent flows, and their solutions often require extensive computational resources.In contrast, non-Newtonian fluid mechanics introduces additional complexities due to the diverse nature of non-Newtonian fluids, including shear-thinning, shear-thickening, and viscoelastic behaviors. Understanding and modeling these complex rheological properties pose significant challenges, particularly when attempting to apply these models to real-world problems, such as the flow of polymers in industrial processes or the behavior of biological fluids in the human body.Experimental challenges also abound in both classical and non-Newtonian fluid mechanics. Obtaining accurate measurements of fluid properties, especially under extreme conditions or in confined geometries, can be technically demanding. Additionally, validating theoretical models with experimental data remains a constant challenge. The interdisciplinary nature of fluid mechanics requires collaboration between researchers from various fields, such as physics, engineering, and biology. Bridging these gaps and fostering effective communication among experts from different domains can be a challenge in itself.

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