Heat and Mass transfer in MHD Viscoelastic Micropolar fluid flow over a stretching sheet in the presence of Thermal Radiation

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Abstract

In the present work, the heat and mass phenomenon in magnetohydrodynamics viscoelastic Micropolar fluid in the presence of thermal radiation is analysed. The fluid with constant thermal conductivity flows over a stretching sheet. The basic equations of the model governing the flow of non-Newtonian fluids occur as partial differential equations that have been reduced to simpler form of ODE by similarity transformation. The Homotopy perturbation method is used to solve the transformed equations.The effects of various physical parameters such as viscoelastic parameter, magnetic parameter, Prandtl number, micropolar parameter, thermal radiation parameter, which determine the velocity, temperature, microrotation and concentration profileare shown in graphs. The results exhibit excellent agreement with the work of present and previous limiting results of other literature cited.

Keywords: Micropolar Fluid; Viscoelastic; Magnetohydrodynamics (MHD); Thermal Radiation; Stretching Sheet

1. Introduction

Recent focus on the study of the heat and mass transfer in MHD micropolar fluid layer in the presence of thermal radiation has received considerable interest, because of its widespread applicability in geothermal power technology, petroleum recovery, glass fibre manufacturing, metal extrusion, wire drawing, hot rolling of steel, cold rolling or drying of papers. The thermal convection in micropolar fluid layer is always important. Some of the chemical fluids having better electromagnetic properties are recommended as their flow can be regulated by applying external magnetic fields. An extreme care must be given in some practical situations to controlthe rate of cooling liquids when the extrudate is stretched and get rapid solidification such as in the process of the drawing, annealing of tinned copper wires, drawing and fabrication of plastic film and artificial fibre, extrusion process of material in which heat-treated materials travel between a feed roll and windup rolls or on conveyer belts system. Therefore in these phenomena the quality of product is based on thermal conductivity of stretching surface. The micropolar fluid model is sufficient to explain the behaviour of special lubricants, liquid crystals with rigid molecule orientation, animal blood, some organic liquids and colloidal or suspension solutions. The significant importance of the microrotation and couple stresses is very useful in micropolar fluids [1], [2]. In chemical engineering and many biochemical situations, the effect of rotation on thermal convection in micropolar fluid is remarkable. Sharma and Kumar[3]studied the effect of uniform rotation of micropolar fluid on thermal instability. They observed that the coupling between thermal and micropolar effects might introduce oscillatory motion in the system. A non-Newtonian fluid formed by a viscous component and an elastic one is the viscoelastic fluid. In short, viscoelastic fluid is the combination of a solvent and some polymer like as paints, biological fluids (e.g., DNA suspension) and polymeric suspension. The viscoelastic fluids are very interesting for industrial applications [4]. The flow of non-Newtonian fluids has a non-linear relationship between the shear stress and shear rate. Viscoelastic fluids have shear-dependent viscosity. Many mathematical models, describing different constitutive equations involving different set of empirical parameters, have been reported in the literature. During the last few decades, many researchers have focused on the boundary layer flow in MHD over a stretching surface due to its wide application in many technological processes such as food processing sector, MHD thermal power generator, the process of solidification of liquid crystal, the boundary layer control in aerodynamics, in metal working process and metallurgy. Further, the heat and mass transfer are an integral part of stretching surface in polymer extrusion, cable coating and chemical reaction etc. Anderson[5] was the first to study the MHD flow over a stretching surface. Mahapatra and Gupta[6] worked on the problem of MHD stagnation point flow past a stretching sheet. Siddheshwar et al. [7] presented the effect of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet. Rajagopal et al. [8] and S. Abel [9] have studied the flow of a viscoelastic fluid over a stretching sheet. Seddeek[10]studied the heat and mass transfer on a stretching sheet with a magnetic field in a viscoelastic fluid flow through a porous medium. Later this work was extended by many researchers. A. Raptis[11], [12] has investigated the radiation of viscoelastic fluid and examined the temperature profile graphically for different values of the radiation parameter.M. S. Abel and

N. Mahesha[9], [13]worked on the heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. Fatunmbiand Adeniyan [14] have investigated the heat and mass transfer in MHD micropolar fluid flow over a stretching sheet and examined the effect of the controlling parameters on the velocity, temperature, micro rotation and concentration profiles M.Sajid and Hayat [15] examined the effect of thermal radiation on the boundary layer flow over an exponentially stretching sheet and reported series solutions for velocity and temperature. Motivated by all these works the main focus of this work is to determine the impact of heat and mass transfer in MHD micropolar fluid in presence of thermal radiation where the heat is transferred in viscoelastic micropolar fluid over a stretching sheet in an applied magnetic field.The effect of various parameters such as Prandtl number, viscoelastic parameter, magnetic parameter, micropolar parameter,thermal radiation parameter is reported and the results are presented through graphs that highlight the velocity, temperature, micro rotation and concentration profiles.

2. Mathematical Formulation

The physical phenomenon consists of a MHD viscoelastic micropolar fluid flow with heat and mass transfer over a stretching sheet in presence of the thermal radiation. The flow over the stretching sheet is two dimensional, steady and incompressible with *u* and *v* component in *x* and *y* directions, respectively as shown in Fig.1.

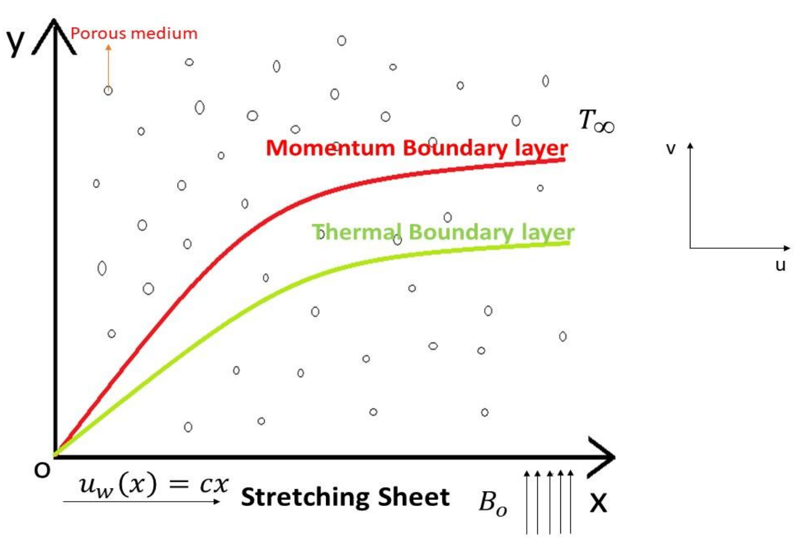

Figure 1: Geometry of fluid flow

The equations for micropolar fluid flow for this study are

$$\frac{\partial u}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\left(1+\frac{K^*}{\mu}\right)\frac{\partial^2 u}{\partial y^2}+\frac{K_0^*}{\rho}\frac{\partial N}{\partial y}-\sigma\frac{B_0^2 u}{\rho}-\frac{K_0}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial^2 u}{\partial y^2}\right)+\mathrm{v}\frac{\partial^3 u}{\partial y^3}\right] \tag{2}$$

$$\rho j\left(u\frac{\partial N}{\partial x}+\mathrm{v}\frac{\partial N}{\partial y}\right)=\gamma\frac{\partial^2 N}{\partial y^2}-K^*\left(2N+\frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}=\frac{K_T}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{4}$$

$$u\frac{\partial C}{\partial x}+\mathrm{v}\frac{\partial C}{\partial y}=D^*\frac{\partial^2 C}{\partial y^2} \tag{5}$$

The appropriate boundary conditions are

$$u=ax, v=0, T=T_w=T_\infty+Ax^s, C=C_w, \quad N=0 \quad at\ y=0 \ \text{(6a)}$$

$$u=0, v=-b\frac{\partial u}{\partial y}=0, T=T_\infty, C=C_\infty, N=0 \quad as\ \ y\to\infty \text{(6b)}$$

Where a>0 and b>0 are constants and s is the wall temperature parameter. There is no free stream velocity, therefore the flow taking place over the stretching of the sheet. Equations (1) & (2) admit a similarity solution,

$$u=\frac{\partial\psi}{\partial y}, \mathrm{v}=-\frac{\partial\psi}{\partial x} \text{ where } \psi=x\sqrt{a\nu}f(\eta)$$

$$\eta=\sqrt{\frac{a}{\nu}}y,\ \ N=a\sqrt{\frac{a}{\nu}}xg(\eta)\ ,\ \ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \ \phi(\eta)=\frac{C-C_\infty}{C_w-C_o} \tag{7}$$

Using above similarity variables, equations (1) to (3) become

$$f'''+\frac{1}{(1+N_1)}[ff''-(f')^2+N_1g'-Mf'-K_0^*\{2f'f'''-(f'')^2-ff^{\mathrm{iv}}]=0 \tag{8}$$

$$\left(1+\frac{N_1}{2}\right)g''+fg'-f'g-N_1(2g+f'')=0 \tag{9}$$

According to Rosseland approximation,

$$q_r = -\frac{4\sigma_0}{3K_1}\frac{\partial T^4}{\partial y} \tag{10}$$

It is assumed that the temperature variation inside flow is such that T^4 may be expanded using the Taylor's series at point T_0 while neglecting higher order terms, we have

$$T^4 = -3T_\infty^4 + 4T_\infty^3 T$$

$$\therefore \frac{\partial q_r}{\partial y} = -\frac{16\sigma_0 T_\infty^3}{3K_1}\frac{\partial^2 T}{\partial y^2} \tag{11}$$

Using similarity variables (7), equations (4), (11) and (5) respectively yield

$$(1+R)\theta'' + P_r(f\theta' - sf'\theta) = 0 \tag{12}$$

$$\phi'' + S_c f\phi' = 0 \tag{13}$$

Where $M = \frac{\sigma B_0^2}{a\rho}$ $K_0^* = \frac{K_0 a}{\mu}$, $N_1 = \frac{K^*}{\mu}, \gamma = \mu\left(1+\frac{N_1}{2}\right)j, j = \frac{\nu}{a}$,

$P_r = \frac{\mu C_p}{K_T}$ is the Prandtl number, $R = \frac{16\sigma_0 T_\infty^3}{3K_1 K_T}$ is the Radiation parameter, $S_c = \frac{\nu}{D^*}$ is the Schmidt number.

The boundary conditions in similarity variables become

$$f'(0) = 1,\ f(0) = 0,\ g(0) = 0,\ \theta(0) = 1,\ \phi(0) = 1 \tag{14a}$$

$$f'(\infty) \to 0,\ f''(\infty) \to 0,\ g(\infty) \to 0,\ \theta(\infty) \to 0, \quad \phi(\infty) \to 0 \tag{14b}$$

4. Physical Quantities of Interest

The physical parameters of most interest are skin fraction coefficientC_f, the local Nusselt numbersNu_x and the local Sherwood numberSh_x, defined by

$$\frac{1}{2}C_f\sqrt{R_{ex}} = (1+N_1)f''(0) \tag{15}$$

$$\frac{N_{ux}}{(1+R)\sqrt{R_{ex}}} = \theta'(0) \tag{16}$$

$$\frac{Sh_x}{\sqrt{R_{ex}}} = -\phi'(0) \quad (17)$$

Where $R_{ex} = \frac{u_w x}{v}$ is the local Reynolds number.

5. Homotopy Perturbation Solution

In view of HPM equations (8), (9), (12) and (13) can be expressed as

$$(1-p)L_f(f-f_o) + p[(1+N_1)f''' + ff'' - (f')^2 + N_1 g' - Mf' - K\{2f'f''' - (f'')^2 - ff^{iv}\} = 0 \quad (18)$$

$$(1-p)L_g(g-g_0) + p\left[\left(1+\frac{N_1}{2}\right)g'' + fg' - f'g - N_1(f''' + 2g)\right] = 0 \quad (19)$$

$$(1-p)L_\theta(\theta-\theta_0) + p[(1+R)\theta'' + P_r(f'\theta - sf'\theta)] = 0 \quad (20)$$

$$(1-p)L_\phi(\theta-\theta_0) + p[\phi'' + S_c f\phi'] = 0 \quad (21)$$

where $L_f = \frac{\partial^3}{\partial\eta^3} + \frac{\partial^2}{\partial\eta^2}, L_g = \frac{\partial^2}{\partial\eta^2} - 1, L_\theta = \frac{\partial^2}{\partial\eta^2} - 1, L_\phi = \frac{\partial^2}{\partial\eta^2} - 1$ and $p\epsilon[1,0]$ is the Homotopy-parameter.Due to the fact that $p\epsilon[1,0]$ is a small parameter, we consider the solutions of equations (18) to (21) as a pawer series in p as defined below

$$f = f_0 + pf_1 + p^2 f_2 + \ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

$$g = g_0 + pg_1 + p^2 g_2 + \ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

$$\phi = \phi_0 + p\phi_1 + p^2\phi_2 + \ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

The approximate solutions of equations (8), (9), (11) and (12) may then be obtained as

$$f = \lim_{p\to 1} f = f_0 + f_1 + f_2 + \ldots\ldots\ldots\ldots\ldots\ldots \quad (26)$$

$$g = \lim_{p\to 1} g = g_0 + g_1 + g_2 + \ldots\ldots\ldots\ldots\ldots\ldots \quad (27)$$

$$\theta = \lim_{p\to 1} \theta = \theta_0 + \theta_1 + \theta_2 + \ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

$$\phi = \lim_{p\to 1} \phi = \phi_0 + \phi_1 + \phi_2 + \ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

According to boundary conditions, assuming that

$$L_f(f) = 0, L_g(g) = 0, L_\theta(\theta) = 0\,, L_\phi(\phi) = 0$$

Substituting equations (22)to (25) into equations (18) to (21), respectively and after some simplification and rearrangements based on of p-terms, we have,

$p^{(0)}$:

$$\left.\begin{array}{c} L_f(f_0) = 0,\ f_0(0) = 0,\ f_0^{'}(0) = 1, \quad f_0^{'}(\infty) = 0, \quad f_0^{''}(\infty) = 0 \\ L_g(g_0) = 0, \quad g_0(0) = 0, g_0(\infty) = 0 \\ L_\theta(\theta_0) = 0, \theta_0(0) = 1, \theta_0(\infty) = 0 \\ L_\phi(\phi_0) = 0,\ \phi_0(0) = 1, \phi_0(\infty) = 0 \end{array}\right\} \quad (30)$$

Solving (29), we have

$$f_0(\eta) = 1 - e^{-\eta}, g_0(\eta) = 0, \theta_0(\eta) = e^{-\eta}\,, \phi_0(\eta) = e^{-\eta} \quad (31)$$

$p^{(1)}$:

$$\left.\begin{array}{c} L_f(f_1) + [(1+N_1)f_0^{'''} + f_0 f_0^{''} - \left(f_0^{'}\right)^2 + N_1 g_0^{'} - M f_0^{'} - k_0^{*}\left\{2f_0^{'}f_0^{'''} - \left(f_0^{''}\right)^2 - f_0 f_0^{IV}\right\} = 0 \\ L_g(g_1) + \left[\left(1 + \frac{N_1}{2}\right) g_0^{''} + f_0 g_0^{'} - f_0^{'} g_0 - N_1(f^{''} + 2g_0)\right] = 0 \\ L_\theta(\theta_1) + \left[(1+R)\theta_0^{''} + P_r\left(f_0\theta_0^{'} - s f_0^{'}\theta_0\right)\right] = 0 \\ L_\phi(\phi_1) + \left[\phi_0^{''} + S_c f_0 \phi_0^{'}\right] = 0 \end{array}\right\} \quad (32)$$

With

$$f_1(0) = 0, \quad f_1^{'}(0) = 0, \quad f_1^{'}(\infty) = 0, \quad f_1^{''}(\infty) = 0$$

$$g_1(0) = 0\,, g_1(\infty) = 0$$

$$\theta_1(0) = 1, \theta_1(\infty) = 0$$

$\phi_1(0) = 0, \phi_1(\infty) = 0$

Solving equation (32) with above boundary conditions, we have

$$f_1(\eta) = (M + K_0^* - N_1)(\eta e^{-\eta} + e^{-\eta} - 1) \tag{33}$$

$$g_1(\eta) = \frac{N_1}{2} n e^{-\eta} \tag{34}$$

$$\theta_1(\eta) = \frac{1}{3}(s-1)P_r(e^{-2\eta} - e^{-\eta}) + \frac{1}{2}(1 + R - P_r) n e^{-\eta} \tag{35}$$

$$\phi_1(\eta) = \frac{1}{2}S_c(e^{-\eta} - e^{-2\eta}) - \frac{1}{2}(S_c - 1)\eta e^{-\eta} \tag{36}$$

In general, for $p^{(j)}$ we have

$$L_f(f_j) - L_f(f_{j-1}) + [(1 + N_1)f_{j-1}^{'''} + \sum_{k=0}^{j-1} f_k f_{j-1-k}^{''} - \sum_{k=0}^{j-1} f_k^{'} f_{j-1-k}^{'}$$

$$+N_1 g_{j-1}^{'} - M f_{j-1}^{'} - K_0^*\{2\sum_{k=0}^{j-1} f_k^{'} f_{j-1-k}^{'v} - \sum_{k=0}^{j-1} f_k^{''} f_{j-1-k}^{''} - \sum_{k=0}^{j-1} f_k f_{j-1-k}^{'v}\}] = 0 \tag{37}$$

$$L_g(g_j) - L_g(g_{j-1}) + \begin{bmatrix} \left(1 + \frac{N_1}{2}\right) g_{j-1}^{''} + \sum_{k=0}^{j-1} f_k g_{j-1-k}^{'} \\ -\sum_{k=0}^{j-1} f_k g_{j-1-k} - N_1\left(f_{j-1}^{''} + 2g_{j-1}\right) \end{bmatrix} = 0 \tag{38}$$

$$L_\theta(\theta_j) - L_\theta(\theta_{j-1}) + \left[1 + \theta_{j-1}^{''} + P_r\left(\sum_{k=0}^{j-1} f_k \theta_{j-1-k}^{'} - \sum_{k=0}^{j-1} f_k^{'} \theta_{j-1-k}\right)\right] = 0 \tag{39}$$

$$L_\phi(\phi_j) - L_\phi(\phi_{j-1}) + \left[\phi_{j-1}^{''} + S_c \sum_{k=0}^{j-1} f_k \phi_{j-1-k}^{'}\right] = 0 \tag{40}$$

With boundary conditions

$f_j(0) = 0, f_j^{'}(0) = 0,\ f_j^{'}(\infty) = 0,\ f_j^{''}(\infty) = 0; g_j(0) = 0, g_j(\infty) = 0;$

$\theta_j(0) = 0, \theta_j(\infty) = 0; \phi_j(0) = 0,\ \phi_j(\infty) = 0$

For $j = 2$, we have

$$f_2(\eta) = \left\{(M + K_0^* - N_1)(1 - 4M + 2N_1) - N_1 + \frac{1}{2}N_1^2\right\}(\eta e^{-\eta} + e^{-\eta} - 1)$$
$$+\{N_1^2 - 2(M + K_0^* - N_1)^2\}\eta^2 e^{-\eta} \tag{41}$$

$$g_2(\eta) = -\frac{1}{6}N_1(e^{-\eta} - e^{-2\eta}) + \frac{1}{16}[5N_1^2 + 4N_1 - 3N_1(M+K_0^*)]\eta e^{-\eta}$$

$$-\tfrac{1}{16}[4N_1(M+K_0^*) - N_1^2]\eta^2 e^{-\eta} \quad (42)$$

$$\theta_2(\eta) = -[(M+K_0^* - N_1)\frac{(10s+13)}{9} - \frac{1}{3}P_r(1+P_r)(s-1) - \frac{1}{18}P_r(1+R)(8s-11)$$

$$-\frac{1}{3}P_r\left(\frac{1}{2}P_r + \frac{1}{3}s(s-1)\right) - \frac{2}{9}(P_r - s)(1+R-P_r) + \frac{1}{24}(s-1)P_r(s-2P_r)]e^{-\eta}$$

$$+\left[-\frac{1}{2}(M+K_0^* - N_1) + \frac{1}{2}(1+R-P_r)\left(-R + \frac{1}{2}P_r - \frac{1}{3}(s-1)P_r\right) - \frac{1}{8}(1+R-P_r)^2\right]\eta e^{-\eta}$$

$$-\frac{1}{8}(1+R-P_r)^2\eta^2 e^{-\eta} + \begin{bmatrix} \frac{1}{9}(M+K_0^* - N_1)(10s+13) - \frac{1}{3}P_r(1+P_r)(s-1) \\ -\frac{1}{18}P_r(1+R)(8s-11) - \frac{1}{6}P_r^2 \\ -\frac{1}{9}s(s-1) - \frac{2}{9}(P_r - s)(1+R-P_r) \end{bmatrix} e^{-2\eta}$$

$$+\tfrac{1}{3}\left[(1+s)(M+K_0^* - N_1) - \tfrac{1}{2}(P_r - s)(1+R-P_r)\right]\eta e^{-2\eta} + \tfrac{1}{24}(s-1)P_r(s-2P_r)e^{-3\eta} \quad (43)$$

$$\phi_2(\eta) = \left[\frac{43}{36}S_c^2 - \frac{25}{18}S_c - \frac{1}{9}S_c(M+K_0^* - N_1)\right]e^{-\eta} - \frac{1}{4}\begin{bmatrix} 2S_c^2 + 2S_c(M+K_0^* - N_1) \\ -\frac{1}{2}(S_c - 1)^2 \end{bmatrix}\eta e^{-\eta}$$

$$+\frac{1}{8}(S_c - 1)^2\eta^2 e^{-\eta} + \frac{1}{9}\left(-10S_c^2 - S_c(M+K_0^* - N_1) + \frac{25}{2}S_c\right)e^{-2\eta}$$

$$+\tfrac{1}{3}[S_c^2 - 2S_c(M+K_0^* - N_1)]\eta e^{-2\eta} - \tfrac{1}{12}S_c^2 e^{-3\eta} \quad (44)$$

And for j=3, we have

$$f_3(\eta) = (2e^{-\eta} - 1)F + \frac{1}{2}(\eta^2 e^{-\eta} + 2\eta e^{-\eta} + 6e^{-\eta} - 4)G$$

$$+\frac{1}{3}(\eta^3 e^{-\eta}+6\eta^2 e^{-\eta}+12\eta e^{-\eta}+24e^{-\eta}-18)H$$

$$+\left(-\tfrac{1}{4}+\tfrac{1}{2}\eta e^{-\eta}-\tfrac{1}{4}e^{-2\eta}\right)I+\left(-\tfrac{1}{4}+\tfrac{3}{4}\eta e^{-\eta}-\tfrac{1}{4}(\eta+2)e^{-2\eta}\right)J$$

$$+\{-\tfrac{3}{8}+\tfrac{7}{4}\eta e^{-\eta}-\tfrac{1}{8}(2\eta^2+8\eta+11)e^{-2\eta}\}K \quad (45)$$

Where

$$F=-\frac{1}{8}(8+16N_1+N_1K_0^*)\left\{(M+K_0^*-N_1)(1-2M+2K_0^*)-\frac{1}{2}N_1^2\right\}$$

$$-\frac{1}{8}(16+8N_1+5N_1K_0^*)\left\{\frac{N_1^2}{2}-(M+K_0^*-N_1)^2\right\}$$

$$-(1-K_0^*)(M+K_0^*-N_1)^2+\tfrac{N_1^2}{12}+\tfrac{N_1}{16}\{3N_1^2-N_1(M+K_0^*)\},$$

$$G=-(M+K_0^*-N_1)\left\{(M+K_0^*-N_1)(1-2M+2K_0^*)-\frac{1}{2}N_1^2\right\}$$

$$-(M+N_1+6K_0^*)\left\{\tfrac{N_1^2}{2}-(M+K_0^*-N_1)^2\right\},$$

$$H=-\tfrac{1}{2}(M-N_1-K_0^*)\left\{\tfrac{N_1^2}{2}-(M+K_0^*-N_1)^2\right\}-\tfrac{N_1}{16}\{4N_1(M+K_0^*)-N_1^2\},$$

$$I=(M+K_0^*-N_1)^2(1+2K_0^*)-\tfrac{1}{3}N_1^2,$$

$$J=8K_0^*\left\{\tfrac{N_1^2}{2}-(M+K_0^*-N_1)^2\right\},$$

$$K=-K_0^*\left\{\tfrac{N_1^2}{2}-(M+K_0^*-N_1)^2\right\}.$$

$$g_3(\eta)=\left[-\frac{13N_1}{144}-\frac{95}{144}N_1^2+\frac{35}{36}(M+K_0^*)N_1\right]e^{-\eta}$$

$$+\left[\frac{N_1}{4}-\frac{13N_1^2}{16}+\frac{27}{128}N_1^3+(M+K_0^*-N_1)\left\{\frac{N_1}{8}+\frac{37}{32}N_1^2+\frac{15N_1M}{6}+\frac{9N_1K_0^*}{8}\right\}\right]\eta e^{-\eta}$$

$$+\begin{bmatrix} -\frac{19N_1^2}{128} - \frac{N_1^3}{32} + (M+K_0^*)\left(-\frac{N_1}{8} + \frac{17N_1^2}{32}\right) \\ +(M+K_0^* - N_1)\frac{N_1}{8}(-3 + 5M + 3K_0^* - N) \end{bmatrix}\eta^2 e^{\eta}$$

$$-\frac{N_1}{12}\left[\frac{3N_1^2}{4} - N_1(M+K_0^*) - (M+K_0^* - N_1)^2\right]\eta^3 e^{-\eta}$$

$$+\left[\frac{5}{36}N_1 + \frac{73N_1^2}{144} - \frac{35}{36}N_1(M+K_0^*)\right]e^{-2\eta} - \frac{1}{6}\left(N_1(M+K_0^*) - \frac{N_1^2}{4}\right)\eta e^{-2\eta} \tag{46}$$

$$\phi_3(\eta) = \left[-\frac{D}{3} - \frac{4E}{9} - \frac{26F_1}{27} - \frac{G_1}{8} - \frac{3H_1}{32}\right]e^{-\eta} + \left[-\frac{A}{2} - \frac{C}{4}\right]\eta e^{-\eta}$$

$$+\left[-\frac{C}{4}\right]\eta^2 e^{-\eta} - \frac{C}{6}\eta^3 e^{-\eta} + \left[\frac{D}{3} + \frac{4E}{9} + \frac{26F_1}{27}\right]e^{-2\eta} + \left[\frac{E}{3} + \frac{24F_1}{27}\right]\eta e^{-2\eta}$$

$$+\frac{F_1}{3}\eta^2 e^{-2\eta} + \left[\frac{G_1}{8} + \frac{3H_1}{32}\right]e^{-3\eta} + \frac{H_1}{8}\eta e^{-3\eta} \tag{47}$$

Where

$$A = -\frac{31}{12}S_c^2 + \frac{1}{9}S_c(M+K_0^* - N_1) + \frac{25}{18}S_c + \frac{61}{36}S_c^3 + \frac{8}{9}S_c^2(M+K_0^* - N_1) - \frac{S_c}{8}(S_c - 1)^2 +$$
$$\frac{1}{2}S_c(S_c - 1)(M+K_0^* - N_1) - S_c\left\{(M+K_0^* - N_1)(1 - 4M + 2N_1) - N_1 + \frac{1}{2}N_1^2\right\},$$

$$B = (1 - S_c)\left\{2S_c^2 + 2S_c(M+K_0^* - N_1) - \frac{1}{2}(S_c - 1)^2\right\} + \frac{43}{36}S_c^3 - \frac{25}{18}S_c^2 - \frac{1}{9}S_c^2(M+K_0^* - N_1)$$

$$-\frac{S_c}{4}(S_c - 1)^2 - \frac{1}{2}S_c(S_c - 1)(M+K_0^* - N_1),$$

$$C = -\frac{1}{8}(S_c - 1)^2 + \frac{S_c}{8}(S_c - 1)^2 = \frac{(S_c - 1)^3}{8},$$

$$D = \frac{227}{18}S_c^2 + \frac{1}{9}S_c(M+K_0^* - N_1) - \frac{175}{18}\frac{1}{9}S_c - \frac{51}{12}S_c^3 - \frac{7}{9}(M+K_0^* - N_1)$$

$$+\frac{S_c}{8}(S_c - 1)^2 + \frac{3}{2}S_c(S_c - 1)(M+K_0^* - N_1)$$

$+S_c(M+K_0^*-N_1)(1-4M+2N_1)-N_1S_c+\frac{1}{2}N_1^2S_c$,

$$E=-\frac{5}{3}S_c^2+S_c\left(\frac{2}{3}N_1+\frac{1}{2}N_1^2\right)-\frac{2}{3}S_c(M+K_0^*-N_1)+\frac{7}{6}S_c^3+\frac{7}{3}S_c^2(M+K_0^*-N_1)$$

$+\frac{1}{8}S_c(S_c-1)^2-S_c(S_c-1)(M+K_0^*-N_1)+S_c(M+K_0^*-N_1)(1-4M+2N_1)$,

$F_1=-\frac{1}{8}S_c(S_c-1)^2-\frac{1}{2}S_c(S_c-1)(M+K_0^*-N_1)+S_cN_1^2-2S_c(M+K_0^*-N_1)^2$,

$G_1=\frac{91}{36}S_c^2+\frac{83}{36}S_c^3-\frac{19}{9}S_c^2(M+K_0^*-N_1)$,

$H_1=-\frac{2}{3}S_c^3+\frac{4}{3}S_c^2-\frac{7}{3}S_c^2(M+K_0^*-N_1)$

Now substituting values of f_i, g_i, θ_i,and ϕ_i for the values of *i*=1,2,3, etc., in equations (26) to (29), we get values of$f(\eta)$, $g(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ as

$$f(\eta) = 1-e^{-\eta}$$
$$+\left\{(M+K_0^*-N_1)+\frac{1}{2}N_1^2-(M+K_0^*-N_1)(1+2M)+F+2G+6H+\frac{1}{2}I\right\}e^{-\eta}$$
$$+\left\{N_1^2-2(M+K_0^*-N_1)^2+\left(\frac{1}{2}G+2H\right)\right\}\eta^2e^{-\eta}+\frac{1}{3}H\eta^3e^{-\eta}$$
$$+\left\{(M+K_0^*-N_1)+\frac{1}{2}N_1^2-(M+K_0^*-N_1)(1+2M)+F+2G+6H\right\}\eta e^{-\eta}-\frac{1}{4}Ie^{-2\eta}$$
$$-\left\{\left(F+2G+6H+\frac{1}{4}I\right)+(M+K_0^*-N_1)+\frac{1}{2}N_1^2-(M+K_0^*-N_1)(1+2M)\right\} \quad (48)$$

Where,

$$F = -\left[N_1(M+K_0^* - N_1)(3K_0^* - 3N_1 - M - 2) - \frac{N_1^3}{2} + \frac{N_1^2}{2} - (2M+1)(M+K_0^* - N_1) + (1+3K_0^*)(M+K_0^* - N_1)^2 - 2K_0^*(M+K_0^* - N_1)(3K_0^* - 3N_1 + M - 1) + 2K_0^* N_1^2 + \frac{5N_1^2}{12} + \frac{N_1}{16}\{-11N_1^2 + 4N_1(M+K_0^*)\}\right]$$

$$G = -\left[N_1^3 - N_1(M+K_0^* - N_1)(3K_0^* - 3N_1 + M - 1) - (1+K_0^*)(M+K_0^* - N_1)^2 + \frac{13}{16}N_1^3 - \frac{N_1^2}{4} - \frac{3N_1^2}{4}(M+K_0^*) + M(M+K_0^* - N_1)(K_0^* - N_1 - M - 1) + K_0^*(M+K_0^* - N_1)(4K_0^* - 4N_1 + 2M - 1) - \frac{3}{2}N_1^2 K_0^*\right]$$

$$H = -\left[\frac{1}{2}(M+K_0^* - N_1)\left\{\frac{N_1^2}{2} - (M+K_0^* - K_0^*)^2\right\} + \frac{N_1}{16}\{4N_1(M+K_0^*) - N_1^2\}\right]$$

$$I = \frac{5N_1^2}{6}(1+2K_0^*)$$

$$g(\eta) = \left[-\frac{41N_1}{144} + \frac{N_1^2}{144} + \frac{11}{36}N_1(M+K_0^*)\right]e^{-\eta} + \left[\frac{11N_1}{36} - \frac{91}{432}N_1^2 - \frac{17}{108}N_1(M+K_0^*)\right]e^{-2\eta} - \frac{1}{48}N_1 e^{-3\eta} + \left[-\frac{51N_1^2}{128} + \frac{N_1}{8} + \frac{3N_1}{32}(M+K_0^*) + \frac{1}{8}(M+K_0^* - N_1)^2 - \frac{N_1}{8}\left\{\frac{-N_1}{4} + \frac{141N_1^2}{16} - \frac{15N_1}{4}(M+K_0^*) - (M+K_0^* - N_1)(4M+3)\right\} + \frac{1}{16}[-11N_1^2 + 4N_1 + 4N_1(M+K_0^*)] + \frac{N_1}{2}\right]\eta e^{-\eta} + \left[\frac{349}{1024}N_1^3 - \frac{N_1^2}{16} - \frac{N_1}{8}(M+K_0^*) + \frac{11N_1^2}{56}(M+K_0^*) + \frac{N_1}{64}(M+K_0^* - N_1)(24 - 31K_0^* + 31N_1) - \frac{1}{16}[4N_1(M+K_0^*) -$$

$N_1^2]\Big]\eta^2 e^{-\eta} - \frac{1}{48}\Big\{\frac{11N_1^2}{4} - 4(M+K_0^* - N_1)^2 - 3N_1(M+K_0^*)\Big\}\eta^3 e^{-\eta} - \frac{1}{6}\Big[N_1(M+K_0^*) - \frac{N_1^2}{4}\Big]\eta e^{-2\eta}$ (49)

$\theta(\eta) = e^{-\eta} + \frac{1}{3}S_c(e^{-\eta} - e^{-2\eta}) - \frac{1}{2}(S_c - 1)\eta e^{-\eta} + \Big[\frac{1}{3}\Big\{P_r(M+K_0^* - N_1) + \frac{1}{2}P_r(1 + R - P_r) - \frac{1}{3}(s-1)P_r(1 + 4R - 3P_r + P_r s)\Big\} + \frac{4}{9}\Big\{(s-1)P_r\Big\{\frac{1}{2}(1 + R - P_r) - (M+K_0^* - N_1)\Big\}\Big\} + \frac{1}{24}(s-1)(s-2)P_r^2\Big]e^{-\eta} + \Big[-\frac{1}{2}\Big\{(1 + R - P_r)\Big\{R - \frac{1}{2}P_r + \frac{1}{3}(s-1)P_r\Big\} - P_r(M+K_0^* - N_1)\Big\} + \frac{1}{8}(1 + R - P_r)^2\Big]\eta e^{-\eta} + \frac{1}{8}(1 + R - P_r)^2\eta^2 e^{-\eta} + \Big[\frac{1}{3}\Big\{P_r(M+K_0^* - N_1) + \frac{1}{2}P_r(1 + R - P_r) - \frac{1}{3}(s-1)P_r(1 + 4R - 3P_r + P_r s)\Big\} + \frac{4}{9}(s-1)P_r\Big\{\frac{1}{2}(1 + R - P_r) - (M+K_0^* - N_1)\Big\}\Big]e^{-2\eta} + \frac{1}{3}(s-1)P_r\Big\{\frac{1}{2}(1 + R - P_r) - (M+K_0^* - N_1)\Big\}\eta e^{-2\eta} + \frac{1}{24}(s-1)(s-2)P_r^2 e^{-3\eta} + \Big(-\frac{A_4}{3} - \frac{4}{9}A_5 - \frac{26}{27}A_6 - \frac{A_7}{8} - \frac{3}{32}A_8 - \frac{1}{15}A_9\Big)e^{-\eta} - \frac{A_1}{2}\eta e^{-\eta} - \frac{A_2}{4}\eta(\eta+1)e^{-\eta} - \frac{A_3}{12}e^{-\eta}\eta(2\eta^2 + 3\eta + 3) + \frac{1}{3}A_4 e^{-2\eta} + \frac{1}{9}A_5(3\eta + 4)e^{-2\eta} + \frac{A_6}{27}(9\eta^2 + 24\eta + 26)e^{-2\eta} + \frac{A_7}{8}e^{-3\eta} + \frac{A_8}{32}(4\eta + 3)e^{-3\eta} + \frac{A_9}{15}e^{-4\eta}$ (50)

Where

$$A_1 = \Big[-AR + (1 + R - P_r)\Big(\frac{C}{3} + \frac{4}{9}D + \frac{E}{8}\Big) - P_r\Big(\frac{C}{3} + \frac{4}{9}D + \frac{E}{8} -\Big) + \frac{1}{6}(s-1)P_r^2(M + K_0^* - N_1)(1 - R + P_r) + P_r\Big(\frac{A}{2} + \frac{B}{4} + \alpha_1 + 2\alpha_2\Big)\Big]$$

$$A_2 = \Big[B + (1 + R)\Big(\frac{A}{2} - \frac{3B}{4}\Big) - P_r\Big(\frac{A}{2} - \frac{B}{4}\Big) + \frac{1}{6}(s-1)P_r^2(M + K_0^* - N_1)(1 + R - P_r)\Big]$$

$$A_3 = \Big[(1 + R - P_r)\frac{1}{4}B\Big]$$

$$A_4 = \Big[C - 4(1 + R)\Big(\frac{C}{3} + \frac{D}{9}\Big) - P_r\Big(-C - D - \frac{E}{8} + \frac{A}{2} + \frac{B}{4}\Big) + \frac{1}{6}(s-1)P_r^2(M + K_0^* - N_1)(-4 + R - P_r) + P_r(\alpha_1 + 2\alpha_2) - sP_r\Big(\frac{C}{3} + \frac{4}{9}D + \frac{E}{8}\Big)\Big]$$

$$A_5 = \Big[-\frac{D}{3}(1 + 4R) + P_r\Big(\frac{A}{2} - \frac{B}{4} + \frac{2D}{3} + \alpha_1 + 2\alpha_2\Big) - sP_r\Big(\frac{A}{2} + \frac{B}{4} + \alpha_1 + \alpha_2\Big)\Big]$$

$$A_6 = \left[-\frac{1}{4}BP_r - \frac{1}{6}(s-1)P_r^2(M+K_0^*-N_1)(1+R-P_r) + \frac{\alpha_2}{2}P_r - \frac{1}{4}BsP_r - \frac{1}{2}(1+R-P_r)s(M+K_0^*-N_1) - \frac{\alpha_2}{2}sP_r\right]$$

$$A_7 = \left[-(1+9R)\frac{E}{8} - P_r\left(\frac{2C}{3}+\frac{5D}{9}-\frac{3E}{8}\right) + sP_r\left(\frac{C}{3}+\frac{4D}{9}\right) - \frac{1}{3}(s-2)(s-1)P_r^2(M+K_0^*-N_1)\right]$$

$$A_8 = \left[\frac{D}{3}(s-2)P_r - \frac{1}{3}(s-2)(s-1)P_r^2(M+K_0^*-N_1)\right]$$

$$A_9 = -\left[(s+3)P_r\frac{E}{8}\right]$$

$\phi(\eta) = e^{-\eta} + \frac{1}{2}S_c(e^{-\eta} - e^{-2\eta}) - \frac{1}{2}(S_c-1)\eta e^{-\eta} + \left[-\frac{7}{36}S_c^2 + \frac{1}{18}S_c + \frac{7}{9}S_c(M+K_0^*-N_1)\right]e^{-\eta} - \frac{1}{24}[(S_c-1)(7S_c+3) - 12S_c(M+K_0^*-N_1)]\eta e^{-\eta} + \frac{1}{8}(S_c-1)^2\eta^2 e^{-\eta} + \frac{1}{18}\left(-5S_c^2 + S_c +\right.$
14ScM+K0∗−N1e−2η+16Sc2−Sc+2ScM+K0∗−N1ηe−2η+112Sc2e−3η
+−D3−4E9−2627F−G8−3H32+I15e−η+−A2−C4ηe−η+−C4η2e−η−C6η3e−η+D3+4E9+
2627Fe−2η+E3+24F27ηe−2η+F3η2e−2η+G8+3H32e−3η+H8ηe−3η+I15e−4η (51)

Where,

$$A = \frac{1}{72}(S_c-1)\{-35S_c^2 + 31S_c + 56S_c(M+K_0^*-N_1)\} + \frac{S_c^2}{6}(M+K_0^*-N_1) + s_c\left\{\frac{N_1^2}{2} - (2M+1)(M+K_0^*-N_1)\right\}$$

$$B = \frac{1}{4}(S_c-1)\left\{\frac{1}{6}(S_c-1)(7S_c+3) - 2S_c(M+K_0^*-N_1)\right\} + \frac{S_c}{4}(S_c-1)^2 - \frac{1}{2}S_c(S_c-1)$$

$$C = -\frac{1}{8}(S_c-1)^3$$

$$D = \frac{29}{24}S_c^3 - \frac{1}{18}S_c^2 - \frac{65}{72}S_c + \frac{31}{8}S_c(M+K_0^* - N_1) - \frac{7}{2}S_c^2(M+K_0^* - N_1) - S_c\left\{\frac{N_1^2}{2} - (2M+1)(M+K_0^* - N_1)\right\}$$

$$E = -\frac{7}{8}S_c^3 - \frac{1}{6}S_c^2 + \frac{25}{24}S_c - \frac{7}{3}S_c(M+K_0^* - N_1) - \frac{1}{6}S_c^2(M+K_0^* - N_1)$$

$$F = -\frac{1}{8}S_c^3 + \frac{3}{4}S_c^2 - \frac{5}{8}S_c - \frac{1}{4}S_c\left\{\frac{N_1^2}{2} - (2M+1)(M+K_0^* - N_1)\right\}$$

$$G = -\frac{35}{36}S_c^3 + \frac{13}{36}S_c^2 + \frac{17}{9}S_c^2(M+K_0^* - N_1)$$

$$H = \frac{1}{3}S_c^3 - \frac{1}{3}S_c^2 + \frac{4}{9}S_c^2(M+K_0^* - N_1)$$

$$I = \frac{2}{5}S_c^3 e^{-4\eta}$$

6. Result and discussion

Transfer of heat and mass in MHD Viscoelastic Micropolar fluid flow over a stretching sheet in the presence of thermal radiation is very important because of its vast applications in various industrial processes. We have solved the obtained governing equations numerically by bvp4c package in MATLAB software and a range of values of the various important parameters has been considered to understand how these parameters affect the process of heat and mass transfer in such systems. Figure 1 shows the physical model of the fluid flow. We have plotted in figures 2 to 16, the fluid velocity, microrotation, temperature profile and mass transfer parameters as a function of similarity variable η for various values of important control parameters (viscoelastic parameter K_0^*, magnetic parameter M, micro-rotation parameter N_1, thermal radiation parameter R, Prandtl number Pr, etc.) to explain their effect. Figs.2-4 show the simulated velocity$f\,'(\eta)$ for various values of K_0^* , M and N_1. As shown in Fig.2, the simulated velocity has significant dependence on the viscoelastic parameter $K_0^*$for the lower values of η and increases with increasing K_0^* . The simulated velocity becomes nearly independent of K_0^* parameter at higher values and becomes very close when η is increased beyond 2. In contrast to viscoelastic parameter, variation of the magnetic parameter M does not have significant effect on the viscosity dependence of simulated velocity, as can be seen in Fig.3. As shown in Fig. 4, the velocity profile is independent of variations in microrotation parameter N_1 when η is higher than 1. However, for lower values of η, the simulated velocity decreases with increasing micro-rotation parameter N_1. Figs. 5-7, illustrate the viscosity dependence of micro-rotation g(η) for various values of η, M and N. As can be seen from Fig.5, for higher values of η, the micro-rotation is independent to the variations in micro-rotation parameter N while micro-rotation g(η) increases significantly with increasing N, for lower values of η. As shown in Fig. 6, the g(η) is almost same for given values of viscoelastic parameter K_0^* when viscosity is lower than 0.1818. The values of micro-rotation increase with increasingK_0^*, for higher values of η and different curves approach to each other when η is raised around 3. Similar to Fig.6, the g(η)) is almost same for given values of magnetic parameter M, but now for somewhat higher values of η =0.4242 (see Fig.7). Forfurther higher values of η, the g(η) increases with increasing magnetic parameter and again different curves approach to each other when η is raised around 3, similar to Fig.6. Viscosity dependence of temperature profile $\theta(\eta)$ for different values of various parameters are depicted in Figs. 8-12. For very low viscosity (close to 0) and high value of viscosity (close to 3), the temperature profile is independent of variations in different parameters.For intermediate values of viscosity, the temperature profile$\theta(\eta)$ increases with increasing radiation parameter R and micro-rotation parameter N_1 (see Figs. 8 and 11). However, increasing Prandtl number P_r, viscoelastic parameter K_0^*, the temperature distribution $\theta(\eta)$ decreases with magnetic parameter M can be seen from Figs. 9, 10 and 12 respectively. Fig.9 reveals the temperature of the fluid for different values of Prandtl number. The thickness of thermal boundary layer decreases with increasing of Prandtl number P_r also an increase in Prandtl number shows the reduction in the thermal conductivity and thus reduces the thermal boundary layer thickness. As shown in Figs. 13, 14 and 16, the viscosity dependence of Mass concentration profile $\varphi(\eta)$ decreases with increasing parameter S_c, viscoelastic parameter K_0^* , and magnetic parameter M, respectively. Physically $S_c$represents a relation with the thickness of mass transfer and hydrodynamic boundary layers. The Schmidt number S_c and Prandtl number P_r correlate with each other in heat dissipation systems. However, Mass transfer parameter or concentration $\varphi(\eta)$ increases with increasing micro-rotation parameter N_1, as shown in Fig. 15. The solutions of resulting problems are compared with the existing results. It is found that the obtained results perform good agreement. Variations of different important physical parameters are also analysed by plotting graphs.

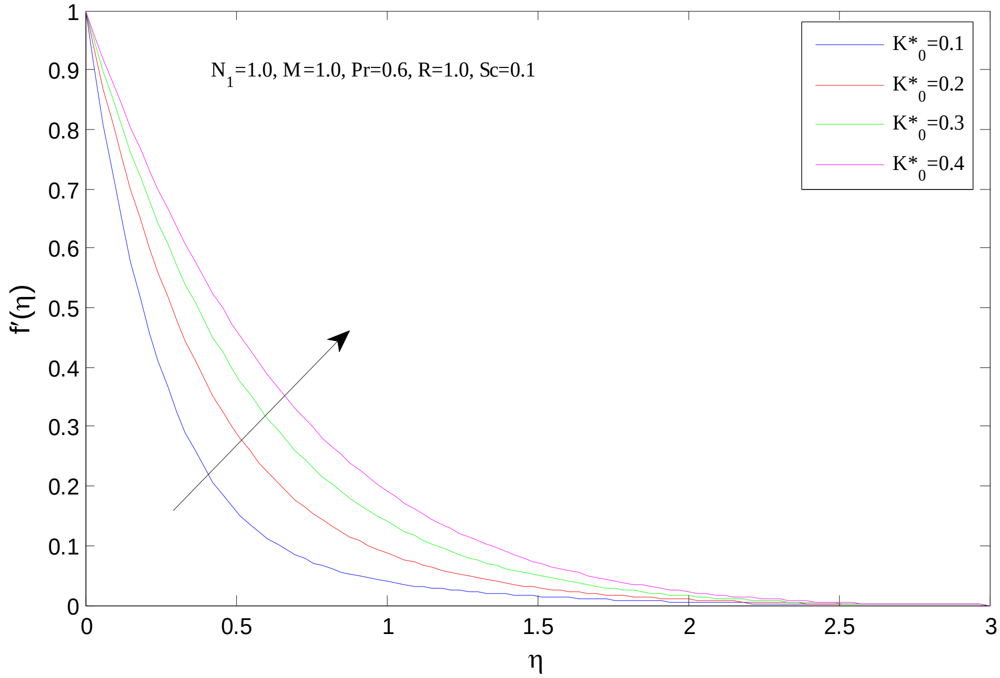

Figure 2: Simulated velocity for various values of viscoelastic parameter

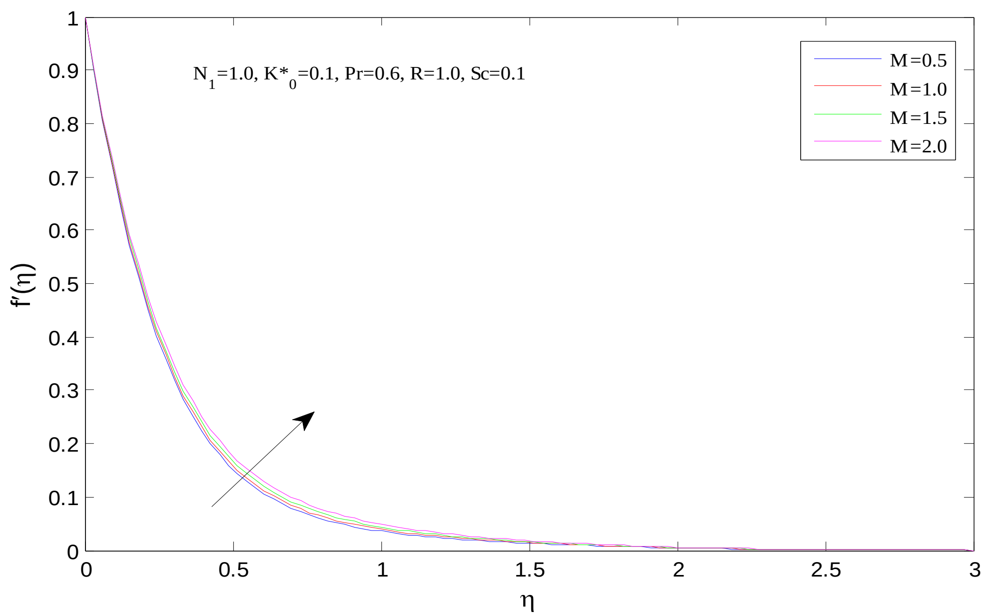

Figure 3: Simulated velocity for various values of magnetic parameter

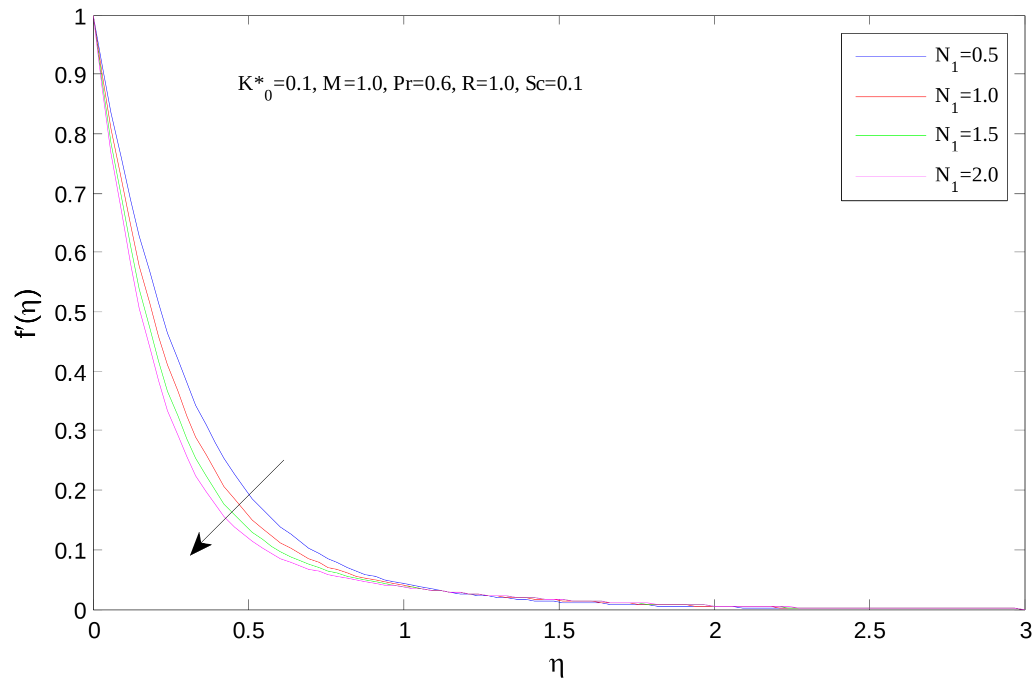

Figure 4: Simulated velocity for different values of micropolar parameter

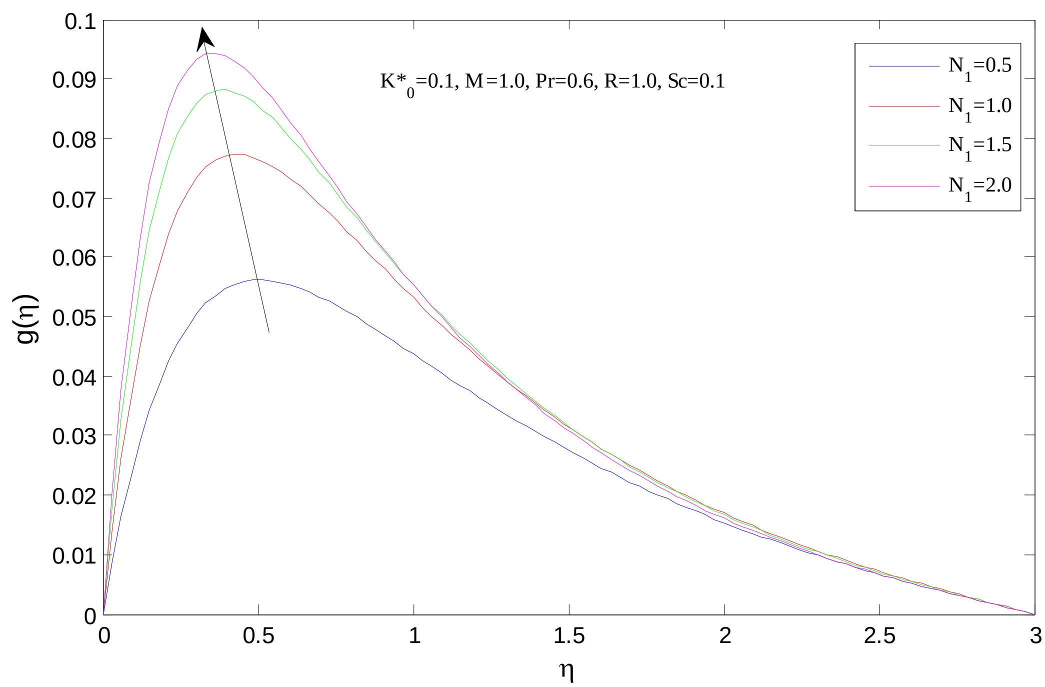

Figure 5: Microrotation for different values of micropolar parameter

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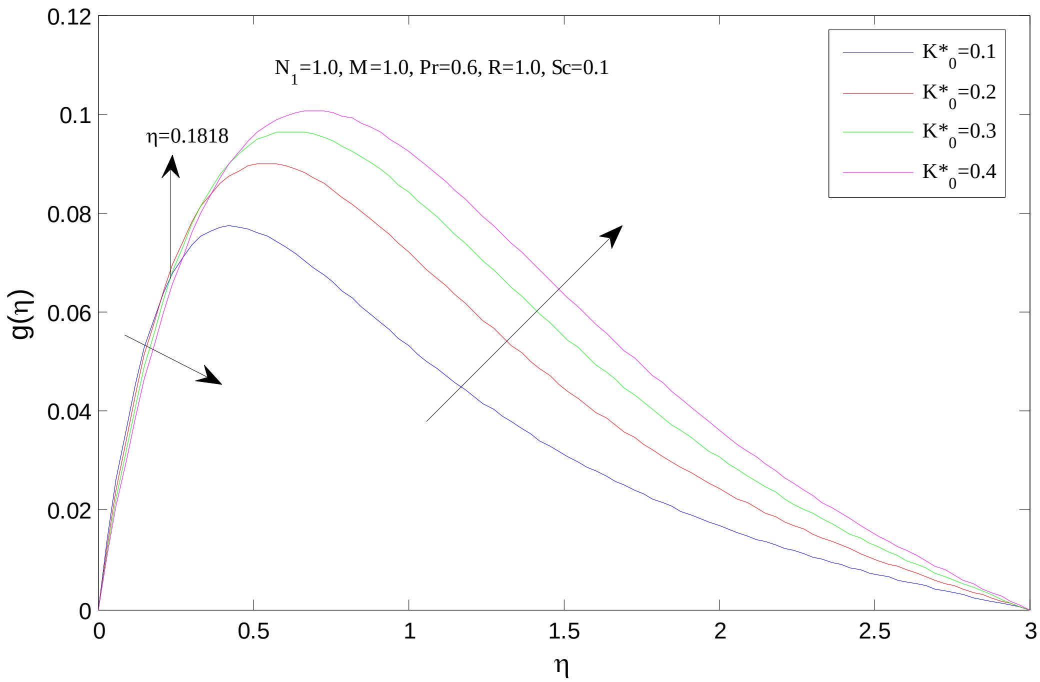

Figure 6: Microrotation for different values of viscoelastic parameter

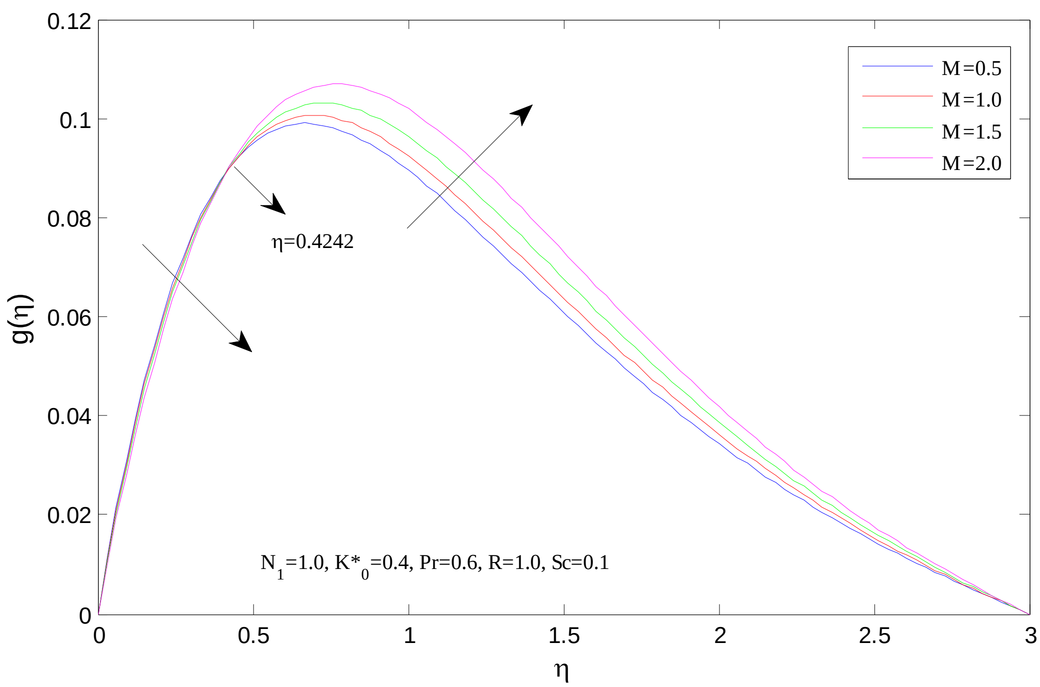

Figure 7: Microrotation for different values of magnetic parameter

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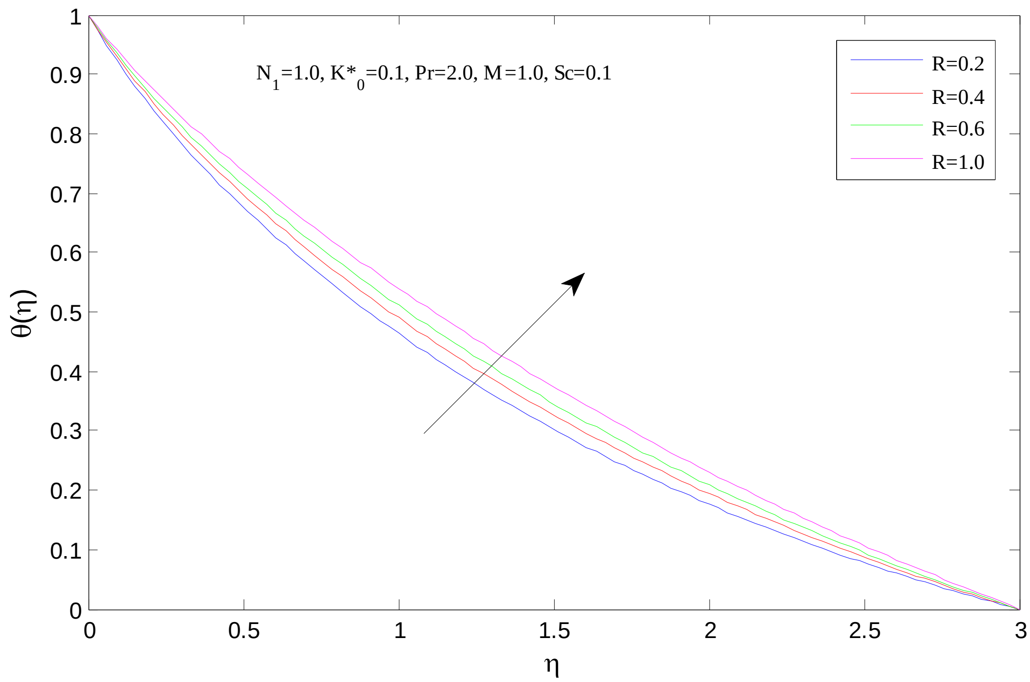

Figure 8: Temperature profile for different values of thermal radiation parameter

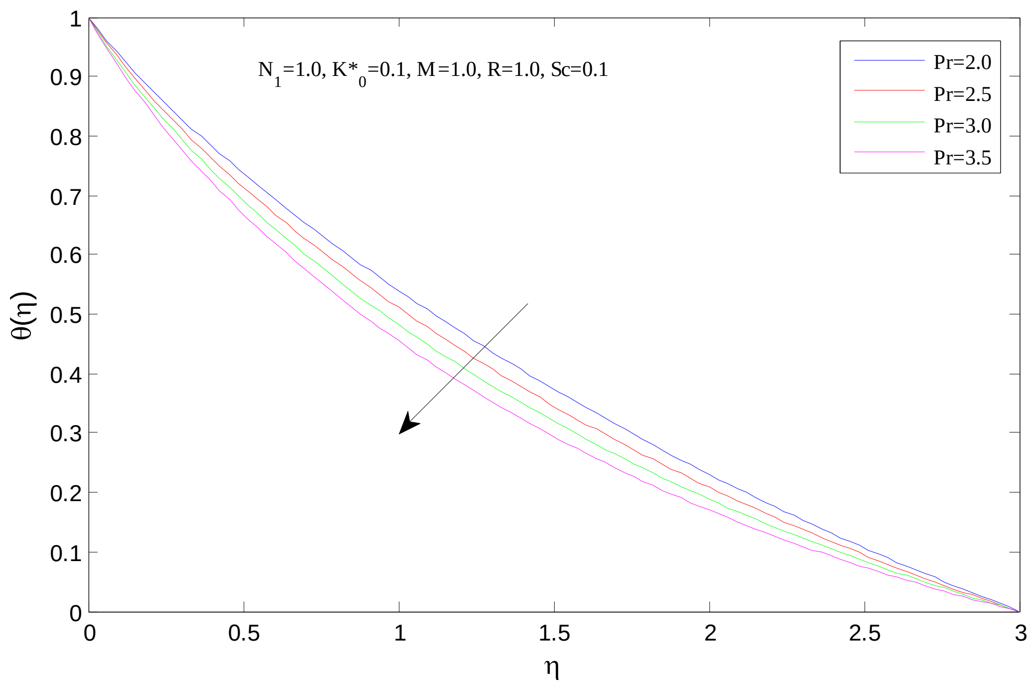

Figure 9: Temperature profile for different values of Prandtl number

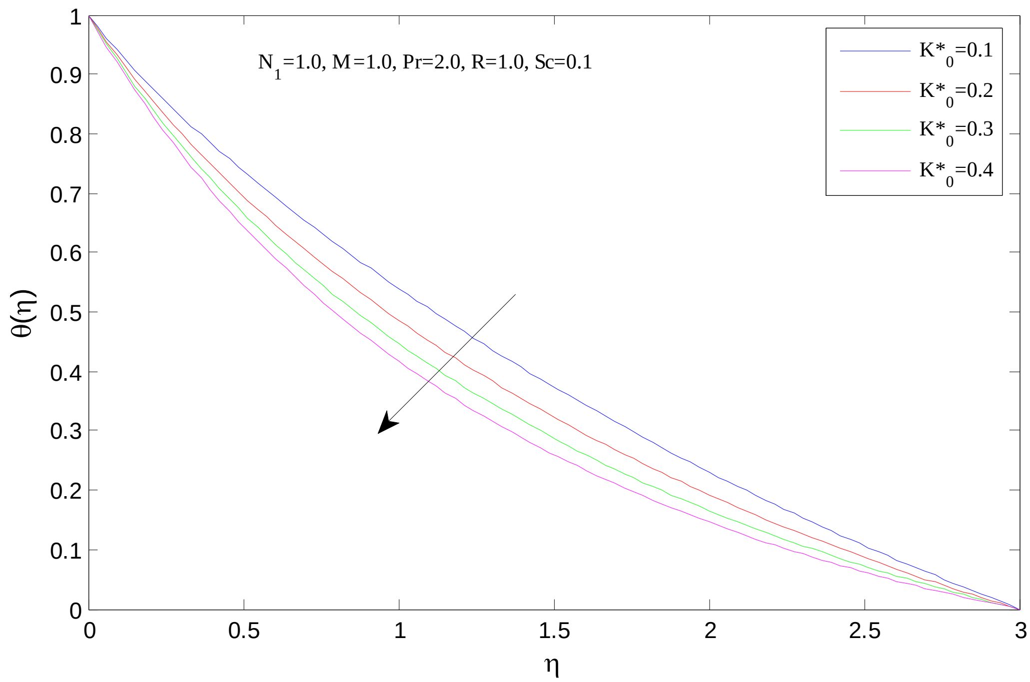

Figure 10: Temperature profile for different values of viscoelastic parameter

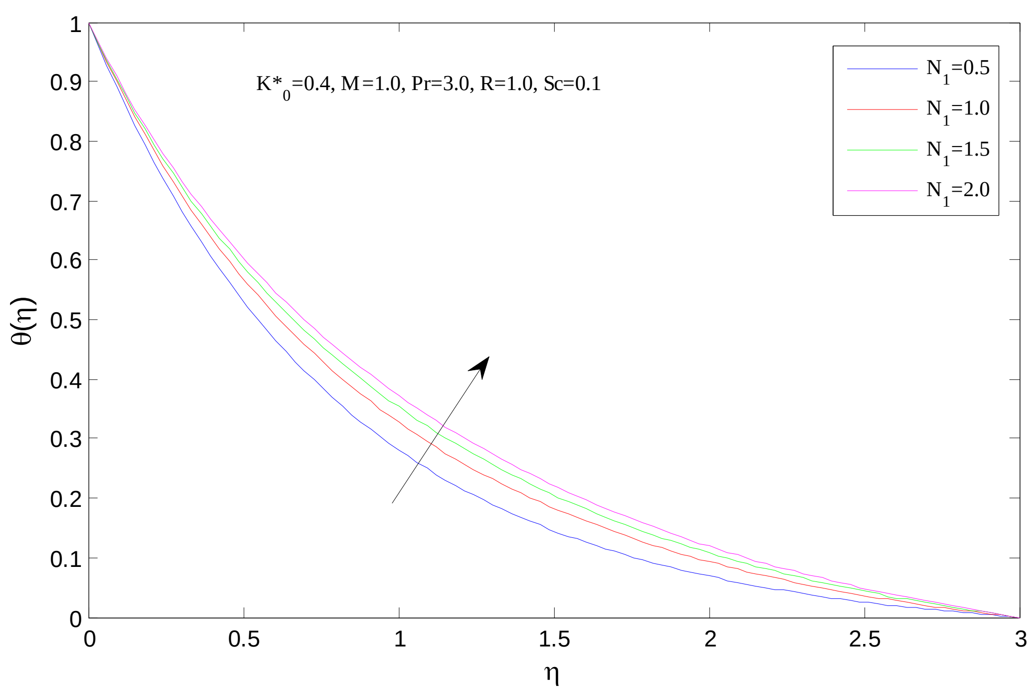

Figure 11: Temperature profile for different values of micropolar parameter

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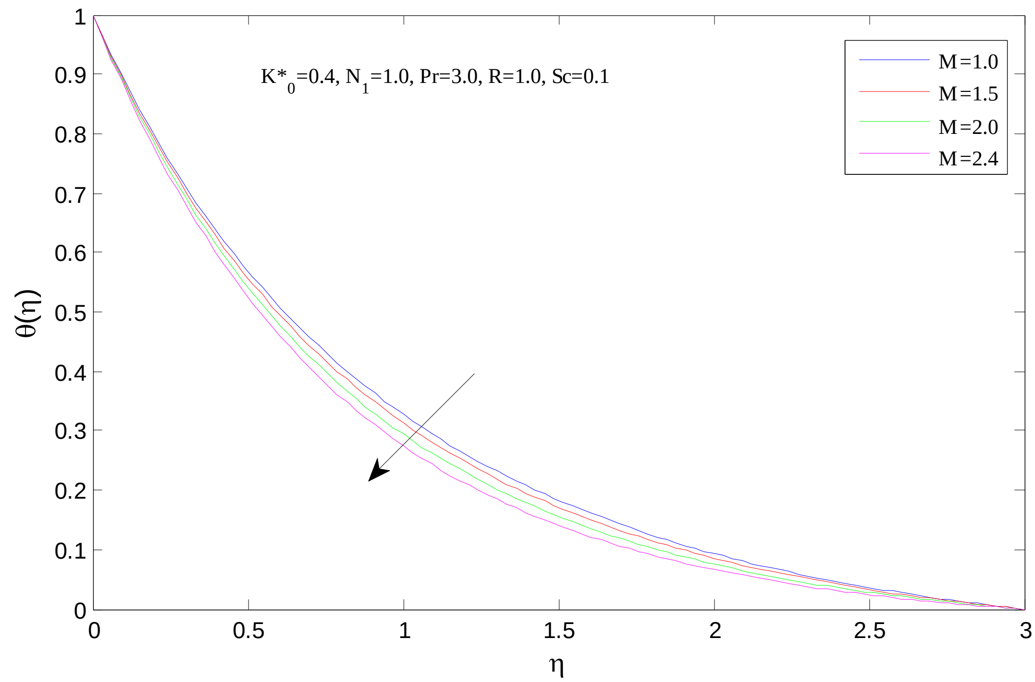

Figure 12: Temperature profile for different values of magnetic parameter

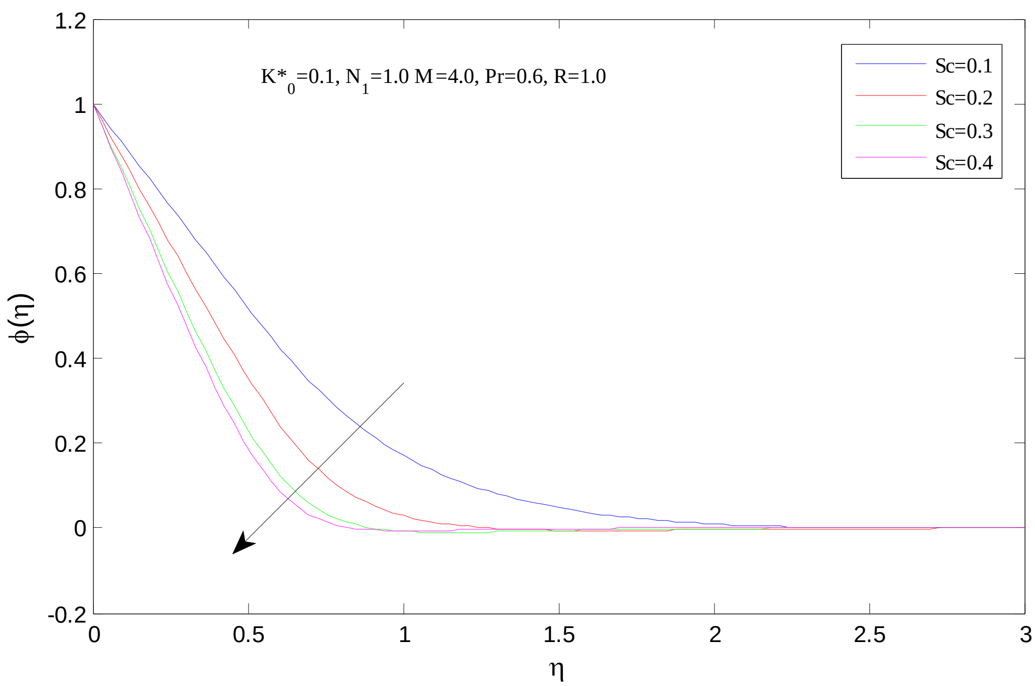

Figure 13: Mass transfer parameter for different values of Schmidt number

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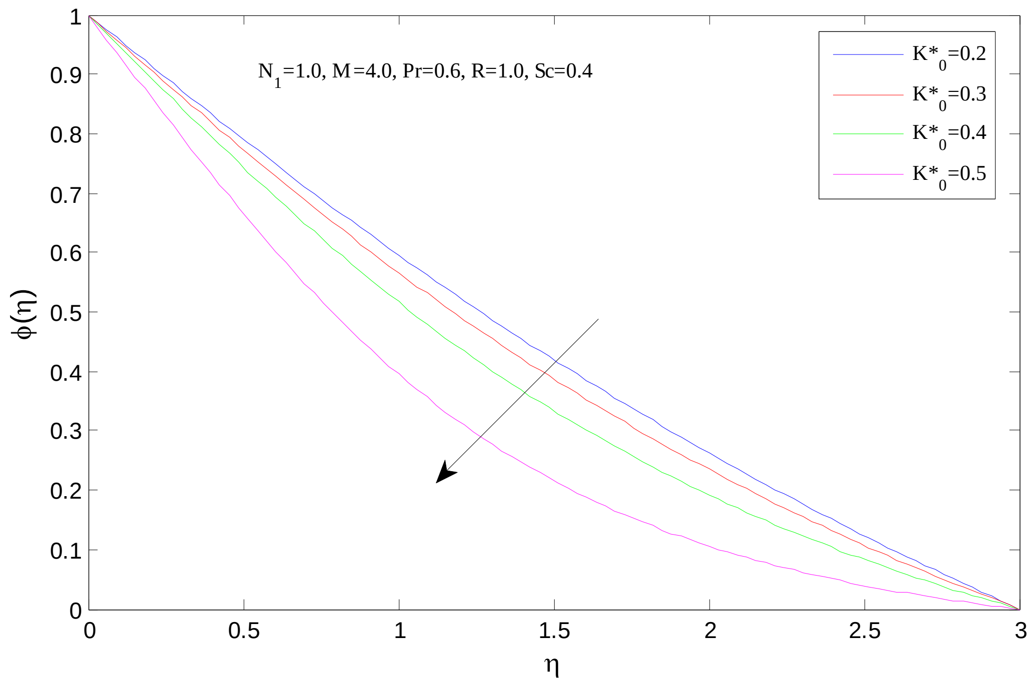

Figure 14: Mass transfer parameter for different values of viscoelastic parameter

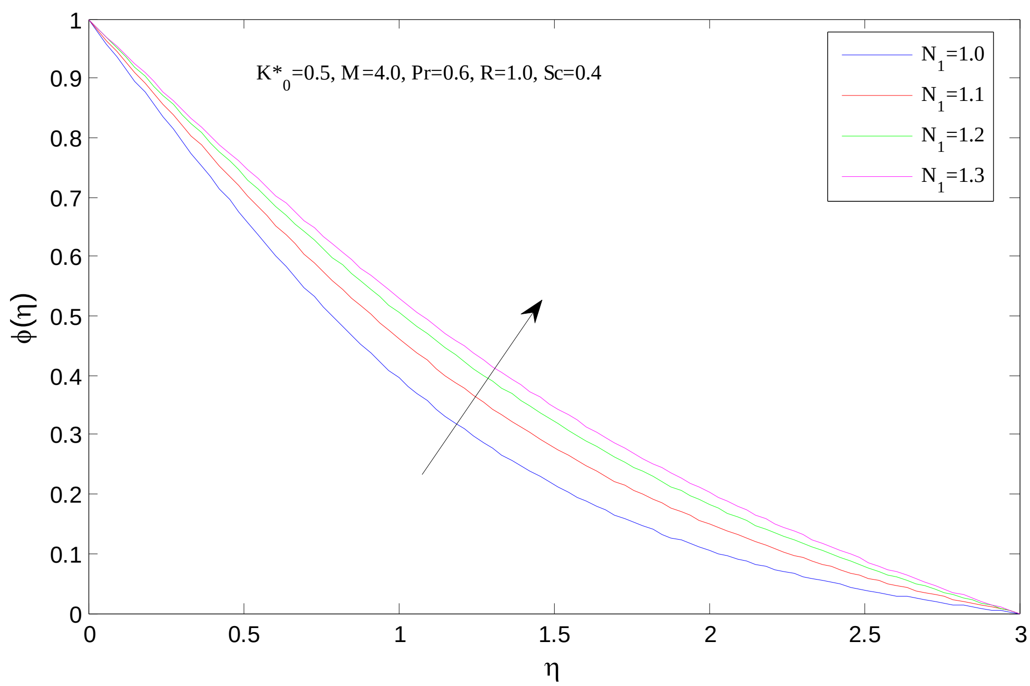

Figure 15: Mass transfer parameter for different values of micropolar parameter

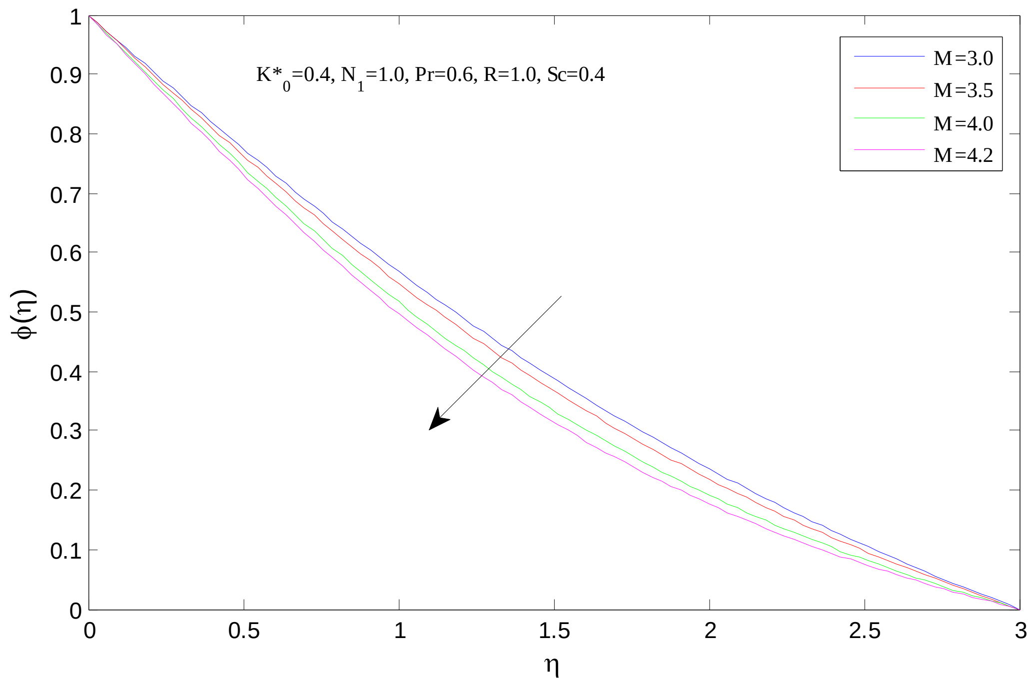

Figure 16: Mass transfer parameter for different values of magnetic parameter

Conclusions

- The change of the magnetic parameter *M and* micro-rotation parameter N_1 does not significantly affect the viscosity dependency of the simulated velocity, in contrast to the viscoelastic parameter K_0^*. The dependency of velocity profile grows as K_0^* rises. At larger η numbers, the simulated velocity practically becomes week dependence on the K_0^* parameter and, as K_0^* exceeds 2, it approaches zero.
- The micro-rotation g(η), for smaller values of η, increases dramatically with increasing *N*, however for larger values of η, the micro-rotation is independent.
- The mass transfer parameter or concentration $\varphi(\eta)$ rises as the micro-rotation parameter N_1 is increased.

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