STATIC PLANE SYMMETRIC INTERACTING FIELDS IN GENERAL THEORY OF RELATIVITY

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Abstract:

We have obtained the plane symmetric space time model in the presence of linearly coupled massless scalar field and source of free electromagnetic field with wet dark fluid by assuming $\alpha = \beta + k_m$. Further we discussed physical and kinematical properties of model. Since $\frac{\sigma}{\theta} \to$ constant as $x \to 0$ and $\frac{\sigma}{\theta} \to \infty$ as $x \to \infty$. As the shear scalar σ is constant at the x at the infinite future, this model is anisotropic in the nature throughout the evolution. Also $\lim_{x\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$, shows that the model does not approach the isotropy for large values of x.

Keywords: Static plane symmetric space time model, Electromagnetic field, Wet dark fluid, Massless scalar field.

1. INTRODUCTION

Plane-symmetric cosmological models are of considerable importance in the context of general relativity as they provide exact solutions to Einstein's field equations. Among these models, the static plane symmetric space-time model assumes significance, particularly when considering thelinear coupling of a massless scalar field, the presence of a free electromagnetic field, and a component representing a mysterious "dark fluid." These models offer a valuable framework for investigating the early stages of the universe's expansion.

In their research, G. Mohanty and U. K. Panigrahi [1] explored the dynamics of a charged perfect fluid and a massive scalar field within a spacetime described by a general axially symmetric Jordan-Ehlers metric, characterized by two degrees of freedom. They also demonstrated that when the meson rest mass and the cosmological constant are both zero, the model reduces to a stiff perfect fluid. They obtained a class of exact solutions by eliminating one of the metric potentials and conducted a thorough investigation into the physical behavior of this model.

In another study, V. Mahurpawar and S. D. Deo [2] examined the Axially symmetric Bianchi-I model, considering cosmic cloud strings as the source coupled with the electromagnetic field within Rosen's bimetric theory of relativity. Their findings indicated that, in this theory, cosmic strings and Maxwell fields did not contribute to the model.

Furthermore, U. K. Panigrahi and R. C. Sahu [3] delved into an anisotropic and homogeneous plane-symmetric cosmological micro-model. They investigated this model in the presence of a massless scalar field within a modified theory of Einstein's general relativity. Additionally, they discussed the physical and geometrical properties of the model, including the presence of singularities. In the jurisdiction of static plane-symmetric space-times, M. Sharif's work [4] involves a classification based on their matter collineations. This classification encompasses scenarios with both degenerate and non-degenerate energy-momentum tensors.Anirudh Pradhan, Purnima Pandey, and Sunil Kumar Singh [5] tackled the Einstein equations for plane-symmetric perfect fluid models. They explored cases involving shear and situations where acceleration vanishes.D. D. Pawar, S. W. Bhaware, and A. G. Deshmukh [6] focused on investigating cosmological models with bulk viscous fluidsin a plane-symmetric dust magnetized string setup. They aimed to establish a deterministic model and analyzed the relationship between metric potentials. Their research delved into the model's behavior in thepresence and absence of a magnetic field while also examining the model's physical and geometrical properties.Vimal Chand Jain, Mahbub Ali, and Raj Bali [7] explored magnetized stiff fluid cylindrically symmetric universes with two degrees of freedom. Their analysis involved perfect fluid distributions and neutral perfect fluids with infinite electrical conductivity. They also discussed various physical aspects of the model in relation to observations.S. N. Bayaskar, D. D. Pawar, and A. G. Deshmukh [8] delved into plane-symmetric models featuring interacting fields. They obtained solutions for two distinct cases: a stifffluid or Zeldovich fluid and disordered radiation. Their research encompassed discussions on the physical and geometric properties of these models.In the field of cosmology, R. Venkateswarlu and K. Pavan Kumar [9] have derived non-static solutions for plane-symmetric cosmologies featuring zero-mass scalar fields within the framework of cosmic strings in general relativity. Their research delved into various physical and geometrical aspects of these models.S. D. Katore, A. Y. Shaikh,and M. M. Sancheti [10] explored plane-symmetric universes with dark energy originating from a wet dark fluid. They obtained exact solutions to the Einstein field equations in a quadrature form, introducing a novel equation of state for the dark energy component of the universe.P. Sharad P. Kandalkar, Amrapali P. Wasnik, and Sunita P. Gawande [11] delved into the study of spherically symmetric cosmological models involving string clouds and string fluids in the presence of an electromagnetic field, all within the framework of general relativity. They found solutions to the Einstein field equations under the assumption of a one-parametric group of conformal motion.D. D. Pawar, R. V. Mapari, and V. M. Raut [12] investigated magnetized strange quark matter using a plane-symmetric cosmological model within the context of Lyra geometry. They obtained results from the solution of field equations, leveraging the relationship between shear scalar and scalar expansion of spacetime. Additionally, they computed dynamical cosmological parameters, the equation of state for strange quark matter, and discussedvarious physical and geometrical properties of the model.In another study, D. D. Pawar, S. P. Shahare, and Y. S. Solanke [13] examined a tilted plane-symmetric cosmological model that incorporates a perfect fluid, heat conduction, and a massless scalar field. They explored analytical solutions of field equations, imposing a power law relation between the metric potentials. Additionally, they discussed several physical and kinematic parameters associated with the model.

Here, we have investigated the static plane symmetric model of interacting fields. We obtained solution by assuming $\alpha = \beta + k_m$ for the static plane symmetric model. Further we discussed physical and kinematical properties of model.

2. THE METRIC AND FIELD EQUATIONS

Consider the metric for static plane symmetric space-time is given by

$$ds^2 = e^{\beta} dt^2 - dx^2 - e^{\alpha}(dy^2 + dz^2) \tag{2.1}$$

Where α and β are functions of x only.

$$\left(T_i^j = T_{i\ EMF}^j + T_{i\ MSF}^j + T_{i\ WDF}^j\right) \tag{2.2}$$

Where, T_i^j is the energy-momentum tensor.

The energy-momentum tensor for an electromagnetic field is defined as

$$T_{i\ EMF}^j = -F_{ir}F^{jr} + \frac{1}{4}F_{ab}F^{ab}g_i^j \tag{2.3}$$

Where $T_{i\ EMF}^j$ is the electromagnetic energy-momentum tensor and F_i^j is the electromagnetic field tensor.

We assume that F_{34} is the only non-vanishing component of F_{ij} which corresponds to the presence of a magnetic field along the z-direction.

Leads to

The non-vanishing components $T_{i\ EMF}^j$ are

$$T_{1\ EMF}^1 = T_{2\ EMF}^2 = -T_{3\ EMF}^3 = -T_{4\ EMF}^4 = -\frac{1}{2}\frac{\omega^2}{e^{\alpha+\beta}} \tag{2.4}$$

Where,

$$F_{34} = \omega = \text{constant} \tag{2.5}$$

The energy-momentum tensor for a massless scalar field is given by

$$T_{i\ MSF}^j = V_{,i}V'^j - \frac{1}{2}g_i^j V_{,s}V'^s \tag{2.6}$$

Where $T_{i\ MSF}^j$ is the energy-momentum tensor for massless scalar field V also satisfy the equation.

$$g^{ij}V_{;ij} = \sigma \tag{2.7}$$

Where σ is the charge density, comma (,) and semicolon (;) denote partial and covariant differentiation respectively.

The non-vanishing components $T_{i\ MSF}^j$ are given by

$$T_{1\ MSF}^1 = T_{2\ MSF}^2 = T_{3\ MSF}^3 = -T_{4\ MSF}^4 = \frac{-\overline{V}^2}{2} \tag{2.8}$$

Also, the energy-momentum tensor $T_{i\ WDF}^{j}$ for wet dark fluid distribution is given by

$$T_{i\ WDF}^{j} = (p_{WDF} + \rho_{WDF})u_i u^j - P_{WDF}\delta_i^j \qquad (2.9)$$

Where p_{WDF} is isotropic pressure and ρ_{WDF} is the matter density and u_i is the flow vector of the fluid is given by,

$$u_i u^j = -1 \qquad (2.10)$$

In a comoving system of coordinates,

$$T_{1\ WDF}^{1} = T_{2\ WDF}^{2} = T_{3\ WDF}^{3} = -p_{WDF} \text{ and } T_{4\ WDF}^{4} = \rho_{WDF} \qquad (2.11)$$

The Einstein field equation in general relativity is

$$G_i^j = R_i^j - \frac{1}{2}Rg_i^j = -8\pi k T_i^j \qquad (2.12)$$

Where R_i^j is known as the Ricci tensor and $R = g^{ij}R_{ij}$ is the Ricci scalar and

With the help of equations (2.1) to (2.12) can be written as

$$\frac{1}{4}\left[2\bar{\alpha}\bar{\beta} + \bar{\alpha}^2\right] = -8\pi k\left[-\frac{\omega^2}{2e^{\alpha+\beta}} - \frac{\bar{V}^2}{2} - p_{WDF}\right] \qquad (2.13)$$

$$\frac{1}{4}\left[2\bar{\bar{\alpha}} + \bar{\alpha}\bar{\beta} + \bar{\alpha}^2 + \bar{\beta}^2 + 2\bar{\bar{\beta}}\right] = -8\pi k\left[-\frac{\omega^2}{2e^{\alpha+\beta}} - \frac{\bar{V}^2}{2} - p_{WDF}\right] \qquad (2.14)$$

$$\frac{1}{4}\left[2\bar{\bar{\alpha}} + \bar{\alpha}\bar{\beta} + \bar{\alpha}^2 + \bar{\beta}^2 + 2\bar{\bar{\beta}}\right] = -8\pi k\left[\frac{\omega^2}{2e^{\alpha+\beta}} - \frac{\bar{V}^2}{2} - P_{WDF}\right] \qquad (2.15)$$

$$\frac{1}{4}\left[4\bar{\bar{\alpha}} + 3\bar{\beta}^2\right] = -8\pi k\left[\frac{\omega^2}{2e^{\alpha+\beta}} + \frac{\bar{V}^2}{2} + \rho_{WDF}\right] \qquad (2.16)$$

Here, the bar denotes the differentiation with respect to x

$$\bar{\alpha} = \frac{\partial\alpha}{\partial x}\ ,\bar{\beta} = \frac{\partial\beta}{\partial x}\ ,\bar{\bar{\alpha}} = \frac{\partial^2\alpha}{\partial x^2}\ ,\ \bar{\bar{\beta}} = \frac{\partial^2\beta}{\partial x^2} \qquad (2.17)$$

We shall determine by the exact solutions of field equations using by assuming

$$p_{WDF} = \rho_{WDF} \qquad (2.18)$$

In equations (2.14) and (2.15) we get

$$F_{34} = \omega = 0 \qquad (2.19)$$

The equation (2.13) to (2.16) reduces to

$$\frac{1}{4}\left[2\bar{\alpha}\bar{\beta} + \bar{\alpha}^2\right] = -8\pi k\left[-\frac{\bar{V}^2}{2} - p_{WDF}\right] \qquad (2.20)$$

$$\frac{1}{4}\left[2\bar{\bar{\alpha}} + \bar{\alpha}\bar{\beta} + \bar{\alpha}^2 + \bar{\beta}^2 + 2\bar{\bar{\beta}}\right] = -8\pi k\left[-\frac{\bar{V}^2}{2} - p_{WDF}\right] \qquad (2.21)$$

$$\frac{1}{4}\left[4\bar{\bar{\alpha}} + 3\bar{\beta}^2\right] = -8\pi k\left[\frac{\bar{V}^2}{2} + p_{WDF}\right] \qquad (2.22)$$

3. SOLUTIONS OF FIELD EQUATIONS

To solve the system of field equations represented by equations (2.20) to (2.22), which involves five unknowns: β, V¯, p_WDF, and ρ_WDF, it is necessary to introduce an additional constraint that relates these parameters. This constraint is essential for obtaining explicit solutions to the system.

We assume a relation between metric potential

$$\alpha = \beta + k_m \tag{3.1}$$

Where k_m is constant

By using equations (3.1) in (2.20) to (2.22) we get

$$\frac{1}{4}\left[3\bar{\beta}^2\right] = -8\pi k\left[-\frac{\bar{V}^2}{2} - p_{WDF}\right] \tag{3.2}$$

$$\frac{1}{4}\left[4\bar{\bar{\beta}} + 3\bar{\beta}^2\right] = -8\pi k\left[-\frac{\bar{V}^2}{2} - p_{WDF}\right] \tag{3.3}$$

$$\frac{1}{4}\left[4\bar{\bar{\beta}} + 3\bar{\beta}^2\right] = -8\pi k\left[\frac{\bar{V}^2}{2} + p_{WDF}\right] \tag{3.4}$$

From equation (3.3) and (3.4) we get

$$\frac{\bar{V}^2}{2} + p_{WDF} = 0 \tag{3.5}$$

By putting this value in equations (3.2) to (3.4) we get

$$\frac{1}{4}\left[3\bar{\beta}^2\right] = 0 \tag{3.6}$$

$$\frac{1}{4}\left[4\bar{\bar{\beta}} + 3\bar{\beta}^2\right] = 0 \tag{3.7}$$

By using equation (3.6) in (3.7) we get

$$\bar{\bar{\beta}} = 0 \tag{3.8}$$

By integrating the above equation, we get

$$\beta = k_1 x + k_2 \tag{3.9}$$

From this

$$\alpha = k_1 x + k_3 \tag{3.10}$$

Where $k_3 = k_2 + k_m$

By using the values of α and β in metric equation we get

$$ds^2 = e^{k_1 x + k_2} dt^2 - dx^2 - e^{k_1 x + k_3}(dy^2 + dz^2) \tag{3.11}$$

Without loss of generality $k_2 = k_3 = 0$

$$ds^2 = -dx^2 - e^{k_1 x}(dy^2 + dz^2 - dt^2) \tag{3.12}$$

4. PHYSICAL AND KINEMATICAL PROPERTIES OF THE MODEL

The scalar expansion θ, Hubble's parameter H and shear scalar σ and volume v is defined as,

$$\theta = U^{\alpha}_{;\alpha} = U^{\alpha}_{,\alpha} + U^{\beta}\left[\Gamma^{\alpha}_{\beta\alpha}\right] \quad (4.1)$$

$$\theta = -\frac{k_1 x}{2} \quad (4.2)$$

$$H = \frac{\theta}{3} = \frac{\bar{V}}{V} \quad (4.3)$$

$$H = -\frac{k_1 x}{6} \quad (4.4)$$

$$v = e^{\frac{1}{18}k_1 x} \quad (4.5)$$

$$\sigma_{ij} = \frac{1}{2}\left[U_{i,j} + U_{j,i}\right] - \frac{\theta}{3}\left[g_{ij} + U_i U_j\right] \quad (4.6)$$

$$\sigma_{11} = 0 \quad (4.7)$$

$$\sigma_{22} = \sigma_{33} = -\sigma_{44} = \frac{1}{6}k_1 x(7e^{k_1 x} + 1) \quad (4.8)$$

Now,

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \quad (4.9)$$

$$\sigma^2 = \frac{1}{12}{k_1}^2 x^2\left[\frac{49}{2} + 7e^{-k_1 x} + \frac{e^{-2k_1 x}}{2}\right] \quad (4.10)$$

$$\frac{\sigma^2}{\theta^2} = \frac{\frac{1}{12}{k_1}^2 x^2\left[\frac{49}{2} + 7e^{-k_1 x} + \frac{e^{-2k_1 x}}{2}\right]}{\frac{{k_1}^2 x^2}{36}} \quad (4.11)$$

$$\frac{\sigma^2}{\theta^2} = \frac{147}{2} + 21e^{-k_1 x} + \frac{3}{2}e^{-2k_1 x} \quad (4.12)$$

Now $\frac{\sigma}{\theta} \to$ constant as $x \to 0$ and $\frac{\sigma}{\theta} \to \infty$ as $x \to \infty$. As the shear scalar σ is constant at the x at the infinite future, this model is anisotropic in the nature throughout the evolution. Also $\lim_{x\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$, shows that the model does not approach the isotropy for large values of x.

The equation (4.2) shows that when $\theta \to 0$ as $x \to 0$ and $\theta \to \infty$ as $x \to \infty$. Thus the universe is expanding with increase of x but the rate of expansion becomes fast as x increases.

The equation (4.10) shows that when $\sigma^2 \to 0$ as $x \to 0$ and $\theta \to \infty$ as $x \to \infty$. This shows rate of change of the shape of the universe becomes fast with increase of x.

5. CONCLUSION

To solve the system of field equations represented by equations (2.20) to (2.22), which involves five unknowns: β, V̄, p_WDF, and ρ_WDF, it is necessary to introduce an additional constraint that relates these parameters. This constraint is essential for obtaining explicit solutions to the system. $\alpha = \beta + k_m$. Further we discussed physical and kinematical properties of model. Since $\frac{\sigma}{\theta} \to$ constant as $x \to 0$ and $\frac{\sigma}{\theta} \to \infty$ as $x \to \infty$. As the shear scalar σ is constant at the x at the infinite future, this model is anisotropic in the nature throughout the evolution. Also $\lim_{x\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$ this shows the model doesn't approach the isotropy for larger values of x. This model shows the rate of change of expansion and shape of the universe is fast with x increases.

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