MAGNETO-HYDRODYNAMIC UNSTEADY FREE CONVECTION FLOW OF A VISCOUS-INCOMPRESSIBLE FLUID PAST A HOT VERTICAL PLATE WITH VARIABLE SUCTION

Dr. Surendra Kumar Prabhakar
Assistant Professor (Mathematics)
Shershah Engineering College, Sasaram
(Dept. of Science, Technology & Technical Education, Govt. of Bihar)

ABSTRACT

In this paper it proposed to study the unsteady convection flow of an incompressible electrically conducting viscous-fluid past a hot vertical plate with variable suction in the presence of a transverse magnetic field. The suction velocity is assumed to vary exponentially with time. The flow phenomena are characterized by the non-dimensional parameters like Prandtl number, Grashoff number, Magnetic number and exponential parameter.

Approximate solutions for velocity, temperature distributions and skin-friction have been obtained. The velocity and temperature distributions have been represented graphically. There follows a comprehensive discussion of the effects of magnetic and suction parameters on the velocity and temperature fields in this paper. A comparison of the present work with the previous works in the subject have been discussed.

KEY – WORDS :

Unsteady, convection, incompressible, viscous, Suction, Skin-friction

1. INTRODUCTION :

A number of research workers have studied the unsteady free convection from an infinite vertical plate. Following Lighthill (1954), Stuart (1955), the oscillatory flow of an incompressible, viscous fluid past an infinite plate with variable suction was represented by Messiha (1966). Stuart and Messiha in their analysis assumed that the plate is stationary and the free stream oscillates in magnitude about a non-zero constant. Soundalgeker (1972), (1976) has studied oscillatory flow with and without a magnetic field past an infinite vertical plate with variable suction. He observed a reverse type of flow when the plate moves in a direction opposite to that of the flow. Pandey (1968) and (1971) studied exponential flow with suction. He observed that by increasing suction the velocity field is increased and there is no back flow near the wall for either exponentially increasing or decreasing small perturbations. Hydromagnetic flow of a viscous incompressible fluid due to unsteady motion of a plate with suction was investigated by Pandey (1972). He found that the velocity decreases with increase in the Hartmann number and that the velocity profile decreases with the increase in the suction parameter, there is no back flow near the wall.

Our aim in this paper is to study the free convection of a viscous incompressible fluid with variable suction over a hot vertical plate moving exponentially with time in its own plane. The temperature is assumed to vary exponentially with time.

The assumption that the suction velocity varies exponentially with time is taken because we want to remove the retarded fluid from the boundary layer as quickly as possible, thereby reducing the separation of the fluid. The effect of the transverse magnetic field on the velocity profile and that of the suction parameter on the temperature field has been investigated in both exponentially increasing and decreasing cases. It has been observed that the velocity profile becomes lower with increase in magnetic parameter, and there is no back flow near the wall. The conclusions have been discussed in conjuction with previous works on the topic. The velocity and temperature distributions have been plotted graphically both for exponentially increasing and decreasing cases.

2. FORMULATION OF THE PROBLEM :

The basic equations of motion, the energy equation and the equation of continuity for the present investigation have been given by Soundalgekar (1972) as :

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = \upsilon'\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) + g'\beta'(T' - T'_\infty) - \frac{\sigma'}{\rho'}B'^2_0 u' \quad \text{........ (2.1)}$$

$$\frac{\partial v'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial v'}{\partial y'} = -\frac{1}{\rho'}\frac{\partial p'}{\partial y'} + \upsilon'\left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial v'}{\partial y'^2}\right) \quad \text{........ (2.2)}$$

$$\frac{\partial T'}{\partial t'} + u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \frac{K'}{\rho' C'_p}\left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right)$$

.... (2.3)

and the equation of continuity is

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \qquad \text{......... (2.4)}$$

The origin of the rectangular reference frame is assumed to be at the lowest point of the plate. ***x'***-axis is supposed to be along the vertical infinite plate in the upward direction and *y'*-axis is taken perpendicular to the plate.

In these equations the velocities u' and *v'* are respectively in *x'*-direction and *y'*-direction, t' the time variable, υ' the kinematic viscosity, ρ' the density, B_0' the external magnetic field, σ' the electrical conductivity, *p'* thee static pressure, *g'* the acceleration due to gravity. *β'* the coefficient of volume expression, C_p' the specific heat at constant pressure, *K'* the thermal conductivity, *T'* the temperature in the boundary layer and T_∞' is the temperature far away from the plate.

Assuming all the physical quantities to be independent of *x'*, the equations (2.1) – (2.4) reduce to

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \upsilon'\frac{\partial^2 u'}{\partial y'^2} + g'(T' - T'_\infty) - \frac{\sigma'}{\rho'}B'^2_0 u' \qquad \text{........ (2.5)}$$

$$\frac{\partial v'}{\partial t'} + v'\frac{\partial v'}{\partial y'} = -\frac{1}{\rho'}\frac{\partial p'}{\partial y'} + \upsilon'\frac{\partial^2 v'}{\partial y'^2} \qquad \text{......... (2.6)}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K'}{\rho' C_p'}\frac{\partial^2 T'}{\partial y'^2} \qquad \text{......... (2.7)}$$

and $$\frac{\partial v'}{\partial y'} = 0 \qquad \text{......... (2.8)}$$

subject to be boundary conditions

$$u = v_0\left[a_0 + \varepsilon\sum_{n'=1}^{\infty} C_{n'}\exp(\pm n't')\right]$$

$$T = T_m^{'}\left[a_0 + \varepsilon\sum_{n'=1}^{\infty} C_{n'}\exp(\pm n't')\right]$$

$$-\varepsilon T_\infty^{'}\sum_{n'=1}^{\infty} C_{n'}\exp(\pm n't') \text{ at } y' = 0$$

and $u' \to 0, T' \to T_\infty^{'}$ as $y' \to \infty$ .... (2.9)

where $T_m^{'}$ is the mean value about which temperature fluctuates, $\varepsilon \leq 1, n'$ is the exponential parameter, a_0 a positive real constant. The series under summation is taken such that $C_{n'}$ decreases as n' increases, so that the convergence of the series remains unaltered.

We assume that the suction velocity varies exponentially with time, so that

$$v' = -v_0^{'}\left[a_0 + \varepsilon A\sum_{n'=1}^{\infty} C_{n'}\exp(n't')\right] \qquad \text{......... (2.10)}$$

where A, a_0 are positive real constants and $v_0^{'}$ a constant mean suction velocity.

Using equation (2.10), equations (2.5) and (2.7) can be written as,

$$\frac{\partial u'}{\partial t'} - v_0^{'}\left[a_0 + \varepsilon A\sum_{n'=1}^{\infty} C_{n'}\exp(n't')\right]\frac{\partial u'}{\partial y'}$$

$$= \upsilon'\frac{\partial^2 u'}{\partial y'^2} + g'p'(T' - T_\infty^{'}) - \frac{\sigma'}{\rho'}B_0^{'2}u' \qquad \text{......... (2.11)}$$

and

$$\frac{\partial T'}{\partial t'} - v_0^{'}\left[a_0 + \varepsilon A\sum_{n'=1}^{\infty} C_n \exp(n't')\right]\frac{\partial T'}{\partial y'} = \frac{K'}{\rho' C_p^{'}}\frac{\partial^2 T'}{\partial y^{'2}}$$

.... (2.12)

Now we introduce the non-dimensional variables

$$y = \frac{y' v_o^{'}}{\upsilon'}, t = \frac{v_0^{'2} t'}{4\upsilon}, n = \frac{4\upsilon' n'}{v_0^{'2}}, u = \frac{u'}{v_o^{'}}$$

$$a_n = \frac{C_{n'} v_0^{'2}}{4\upsilon'}, \theta = \frac{T' - T_\infty^{'}}{T_m^{'} - T_\infty^{'}}, P = \frac{n'.C_p^{'}}{K'} \qquad \text{.......... (2.13)}$$

$$G = \upsilon' g' \beta' \left(\frac{T_m^{'} - T_\infty^{'}}{\upsilon^{'3}}\right), M = \frac{4\upsilon' \sigma' B_0^{'2}}{\upsilon_0^{'2} p'}$$

so that equations (2.11) and (2.12) are transformed into

$$\frac{\partial^2 u}{\partial y^2} + \left[a_0 + \varepsilon A\sum_{n=1}^{\infty} a_n \exp(nt)\right]\frac{\partial u}{\partial y} - \frac{1}{4}\frac{\partial u}{\partial y} - \frac{1}{4}M = -G\theta$$

.... (2.14)

and

$$\frac{\partial^2 \theta}{\partial t^2} + P\left[a_0 + \varepsilon A\sum_{n=1}^{\infty} a_n \exp(nt)\right]\frac{\partial \theta}{\partial y} - \frac{1}{4}P\frac{\partial \theta}{\partial t} = 0 \qquad \text{.......... (2.15)}$$

Boundary conditions reduces to

$$u = a_0 + \varepsilon\sum_{n=1}^{\infty} a_n \exp(nt)$$

$$\theta = a_0 + \varepsilon\sum_{n=1}^{\infty} a_n \exp(\pm nt), \text{ at } y = 0 \qquad \text{........ (2.16)}$$

and $u \to 0, \theta \to 0$ as $y \to \infty$

3. SOLUTION OF THE PROBLEM :

We solve equations (2.14) and (2.15) under the boundary conditions (2.16).

Let us supose

$$u(y,t) = p(y) + \varepsilon \sum_{n=1}^{\infty} a_n \exp(\pm nt) q(y) \qquad \text{....... (3.17)}$$

and

$$\theta(y,t) = a_0 + \varepsilon \sum_{n=1}^{\infty} a_n \exp(\pm nt) - \theta_1(y)$$

$$- \varepsilon \sum_{n=1}^{\infty} a_n \exp(\pm nt) \theta_2(y) \qquad \text{........ (3.18)}$$

Substituting u and θ from (3.17) and (3.18) into the equations (2.14) and (2.15), comparing the harmonic terms, and neglecting the coefficients of ε^2, we obtain

$$p'' + a_0 p' - \frac{1}{4} M p = -G a_0 + G\theta_1 \qquad \text{....... (3.19)}$$

$$q'' + a_0 q' - \frac{1}{4}(M \pm n) q + A p' = G\theta_2 - G \qquad \text{........ (3.20)}$$

$$\theta_1'' + P a_0 \theta_1'' = 0 \qquad \text{....... (3.21)}$$

$$\theta_2'' + P a_0 \theta_2' \pm \frac{1}{4} n P \theta_2 + P A \theta_1' \pm \frac{1}{4} n P = 0 \qquad \text{....... (3.22)}$$

where prime denote differentiation with respect of y. The boundary conditions on *p*, *q*, θ_1 and θ_2 are

$$p = a_0, q = 1, \theta_2 = \theta_1 = 0 \text{ for } y = 0$$

$$p \to 0, \theta_1 \to 1, \theta_2 \to 1 \text{ for } y \to \infty \qquad \text{....... (3.23)}$$

Solving equation (3.21) and (3.22) taking into account thee condition (3.23) we find

$$\theta_1 = [1 - \exp(-P a_0 y)] \qquad \text{........ (3.24)}$$

for exponentially increasing and decreasing cases.

$$\theta_2 = 1 \pm \frac{4Pa_0 y}{n}\exp(-Pa_0 y) - \left(1 \pm \frac{4Pa_0 A}{n}\right)\exp\left(-Pa_0 Ly\right)$$

.... (3.25)

Again we solve equations (3.19) and (3.20), using the equations (3.23), (3.24) and (3.25), we get for both the cases

$$p = 4G\left(\frac{1-a_0}{M}\right)\exp(-a_0\mu y)$$
$$+\left(a_0 + \frac{G}{\lambda}\right)\exp\left(-a_0\mu_1 y\right) - 4G\left(\frac{1-a_0}{M}\right) - \frac{G}{\lambda}\exp(-Pa_0 y)$$

.... (3.26)

and

$$q = \exp(-Na_0 y) \pm \frac{A_1}{A_2}[\exp(-Pa_0 y) - \exp(-Na_0 y)]$$
$$-\frac{G}{A_3}\left[\exp(-Pa_0 Ly) - \exp(-Na_0 y)\right]$$
$$\pm\frac{A_1}{A_2}\left[\exp(-Pa_0 Ly) - \exp(-Na_0 y)\right]$$
$$+A_4\left[\exp(-a_0\mu y) - \exp(-Na_0 y)\right]$$
$$+A_5\left[\exp(-a_0\mu_1 y) - \exp(-Na_0 y)\right]$$
$$-\frac{A_6}{\lambda A_2}\left[\exp(-Pa_0 y) - \exp(-Na_0 y)\right]$$

.... (3.27)

where

$$L = \frac{1}{2}\left[1 + \left(1 \pm \frac{n}{Pa_0^2}\right)^{1/2}\right], \mu = \frac{1}{2}\left[1 - \left(1 - \frac{M}{a_0^2}\right)^{1/2}\right],$$

$$\mu_1=\frac{1}{2}\left[1+\left(1+\frac{M}{a_0^2}\right)^{1/2}\right],\lambda=\left[P^2a_0^2-Pa_0^2-\frac{M}{4}\right],$$

$$N=\frac{1}{2}\left[1+\left(1+\frac{M+n}{a_0^2}\right)^{1/2}\right],A_1=\frac{4Pa_0GA}{n},$$

$$A_2=\left[P^2a_0^2-Pa_0^2-\frac{M\pm n}{4}\right],A_3=\left[P^2a_0^2L^2-Pa_0^2L-\frac{M+n}{4}\right]$$

$$A_4=\left[\frac{4GAa_0\mu\left(1-\frac{a_0}{M}\right)}{a_0^2\mu^2-a_0^2\mu-\left(\frac{M+n}{4}\right)}\right],A_5\left[\frac{Aa_0\mu_1\left(a_0+\frac{G}{A}\right)}{a_0^2\mu_1^2-a_0^2\mu_1-\left(\frac{M+n}{a_0^2}\right)}\right]$$

$$\mathrm{A}_6=\mathrm{Pa}_0\mathrm{AG}.$$

From equation (3.17) and (3.26), (3.27) the velocity field is given by

$$u(y,t)=4G\left(\frac{1-a_0}{M}\right)\exp(-a_0\mu y)+\left(a_0+\frac{G}{\lambda}\right)\exp(-a_0\mu_1 y)$$

$$-4G\left(\frac{1-a_0}{M}\right)-\frac{G}{\lambda}\exp(-Pa_o y)$$

$$+\varepsilon\sum_{n=1}^{\infty}a_n\exp(\pm nt)\left[\exp\left(-Na_0y\right)\pm\frac{A_1}{A_2}\{\exp\left(-Pa_0y\right)-\right.$$

$$\exp(-Na_0y)-\frac{G}{A_3}\{\exp(-Pa_0Ly)-\exp(-Na_0y)\}$$

$$\pm\frac{A_1}{A_2}\{\exp(-Pa_0Ly)-\exp(-Na_0y)\}$$

$$+A_4\{\exp(-a_0\mu y)-\exp(-Na_0y)\}$$

$$+A_5\{\exp(-a_0\mu_1y)-\exp(-Na_0y)\}$$

$$- \frac{A_6}{\lambda A_2}\{\exp(-Pa_0 y) - \exp(-Na_0 y)\}\Bigg]$$

.... (3.28)

Again from equations (3.18), (3.24) and (3.25), the temperature distribution is obtained as

$$\theta(y,t) = a_0 + \varepsilon \sum_{n=1}^{\infty} a_n \exp(\pm nt) - [1 - \exp(-Pa_0 y)]$$

$$- \varepsilon \sum_{n=1}^{\infty} a_n \exp(\pm nt)\left[1 \pm \frac{4Pa_0 y}{n} \exp(-Pa_0 y)\right.$$

$$\left. - \left(1 \pm \frac{4Pa_0 A}{n}\right) \exp(-Pa_0 Ly)\right] \quad \text{....... (3.29)}$$

The expression for skin-friction in non-dimensional form is given by

$$\tau_0 = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= A_9 - A_8 - A_7 + \varepsilon \sum_{n=1}^{\infty} a_n \exp(\pm nt)\left[\pm \frac{A_1}{A_2}(Na_0 - Pa_0)\right.$$

$$- \frac{G}{A_3}(Na_0 - Pa_0 L) \pm \frac{A_1}{A_3}(Na_0 - Pa_0 L) + A_4(Na_0 - a_0 \mu)$$

$$\left. + \mathrm{A}_5(\mathrm{Na}_0 - \mathrm{a}_0 \mu_1) - \left\{\frac{\mathrm{A}_6}{\lambda \mathrm{A}_2}(\mathrm{Na}_0 - \mathrm{Pa}_0) - \mathrm{Na}_0\right\}\right]$$

...... (3.30)

where

$$A_7 = a_0 \mu \left(a_0 + \frac{G}{\lambda}\right),$$

$$A_8 = 4G\left(\frac{1 - a_0}{M}\right) a_0 \mu,$$

$$A_9 = G\left[\frac{Pa_0}{\lambda} - \frac{4}{M}(1-a_0)\right]$$

where the upper and lower signs in $\pm$ or μ refer to exponentially increasing and decreasing cases respectively.

4. GRAPHICAL DISCUSSIONS AND CONCLUSIONS :

From equation (3.28) graphs have been plotted in fig.1 and fig. 2 to show the relation between velocity distribution *u(y, t)* and *y*. In fig. 1 the exponentially increasing case of velocity *u(y, t)* against *y* has been plotted and in fig. 2 exponentially decreasing case of velocity *u(y, t)* against *y* has been drawn graphically.

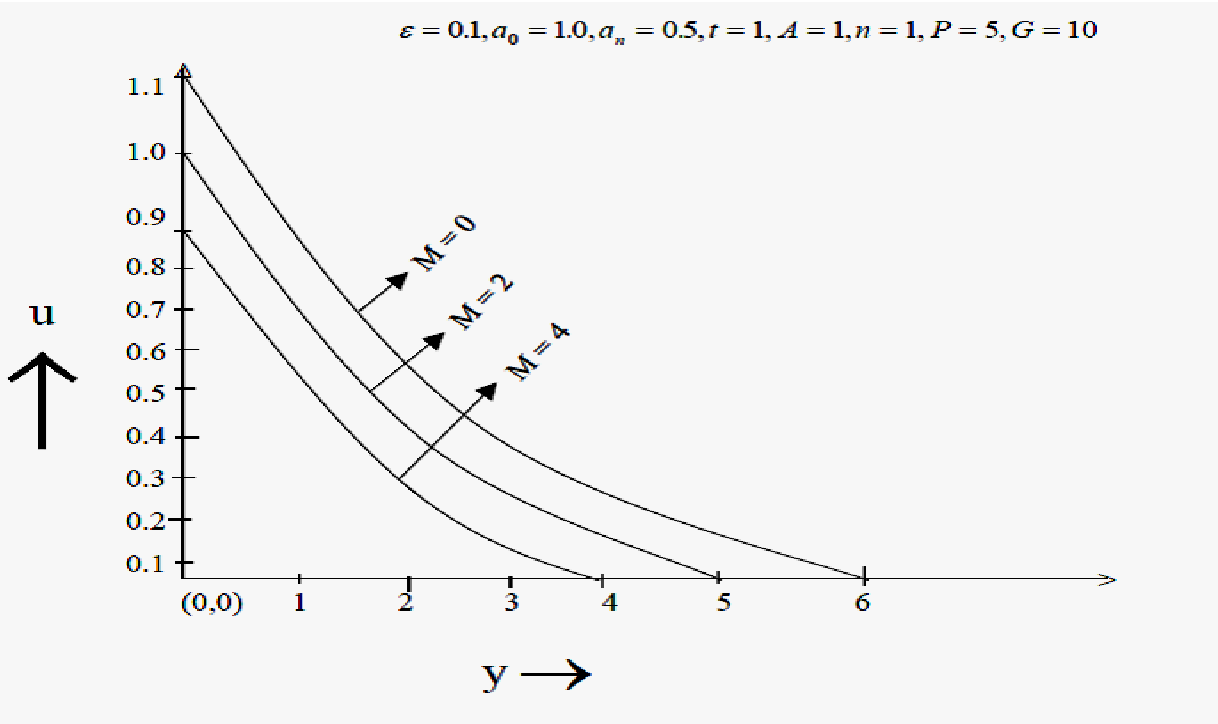

Fig. 1: Exponentially increasing case : Velocity u against y

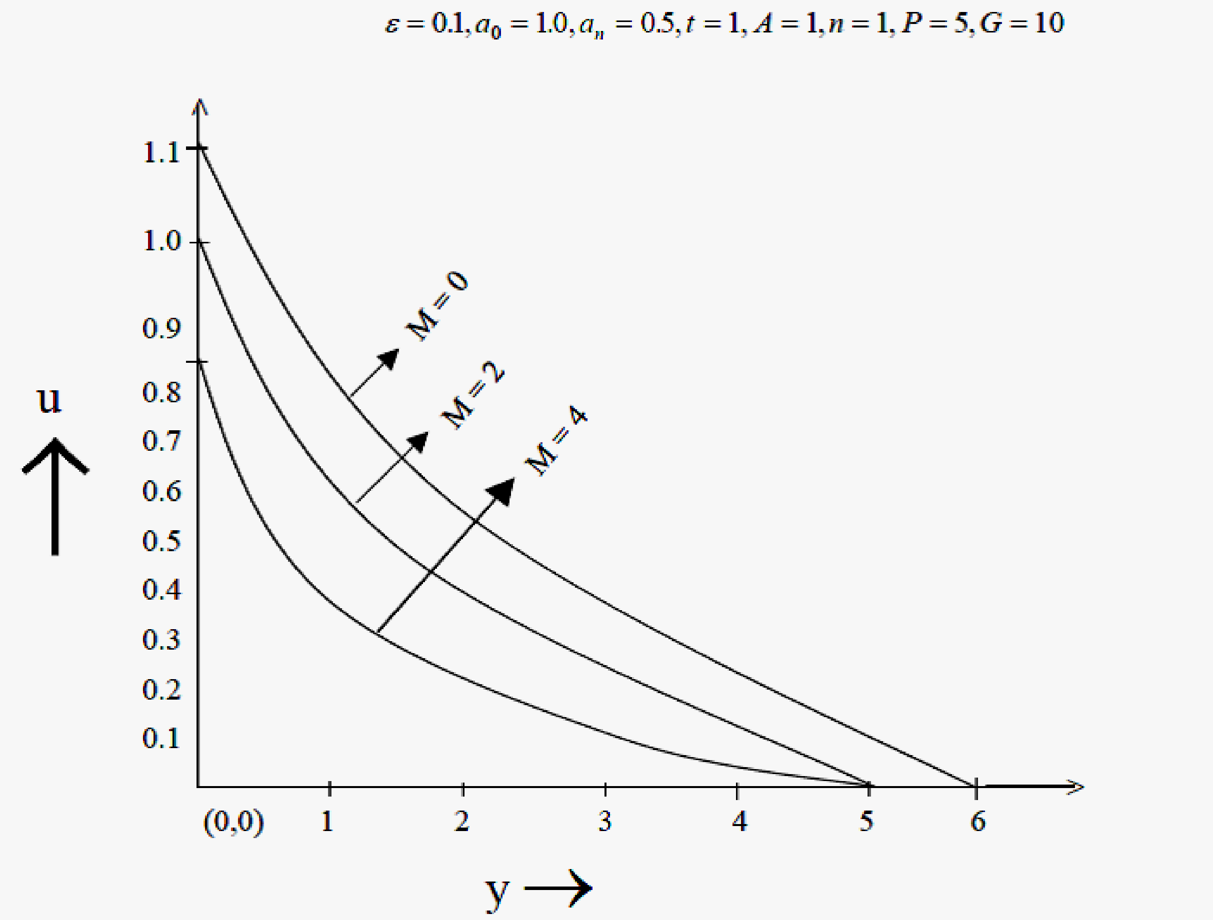

Fig. 2 : Exponentially decreasing case : Velocity u against y

Here we have taken $a_0 = 1, \varepsilon = 0.1, n = 1, t = l$, $a_n = 0.5$, $A = 1, P = 5$ and G = 10. We have taken magnetic parameter M = 0, 2, 4 for both exponentially increasing and decreasing cases of velocity distribution against y. It is observed that the velocity $u(y, t)$ decreases with increase in magnetic parameter M in both exponentially increasing and decreasing cases. A transverse magnetic field reduces the boundary layer thickness and there is no back flow near the wall in either case.

Figs. (3) and (4) have been plotted by using equation (3.29) to show the temperature distribution θ against y. In this context we have taken $a_0 = 1, \varepsilon = 0.1, n = 1, t = l, a_n = 0.5, P = 5, G = 10$. We have plotted temperature θ against y by taking suction parameter A = 0,1, 2 for both exponentially increasing and decreasing cases. It is observed that the temperature θ decreases with the increase in suction parameter A in either cases. It is also observed that there is no effect of magnetic parameter M on the temperature field, probably due to neglecting thee induced magnetic field. It is obvious from the figures that the boundary layer contracts with increasing suction velocity and there is no back flow near the wall.

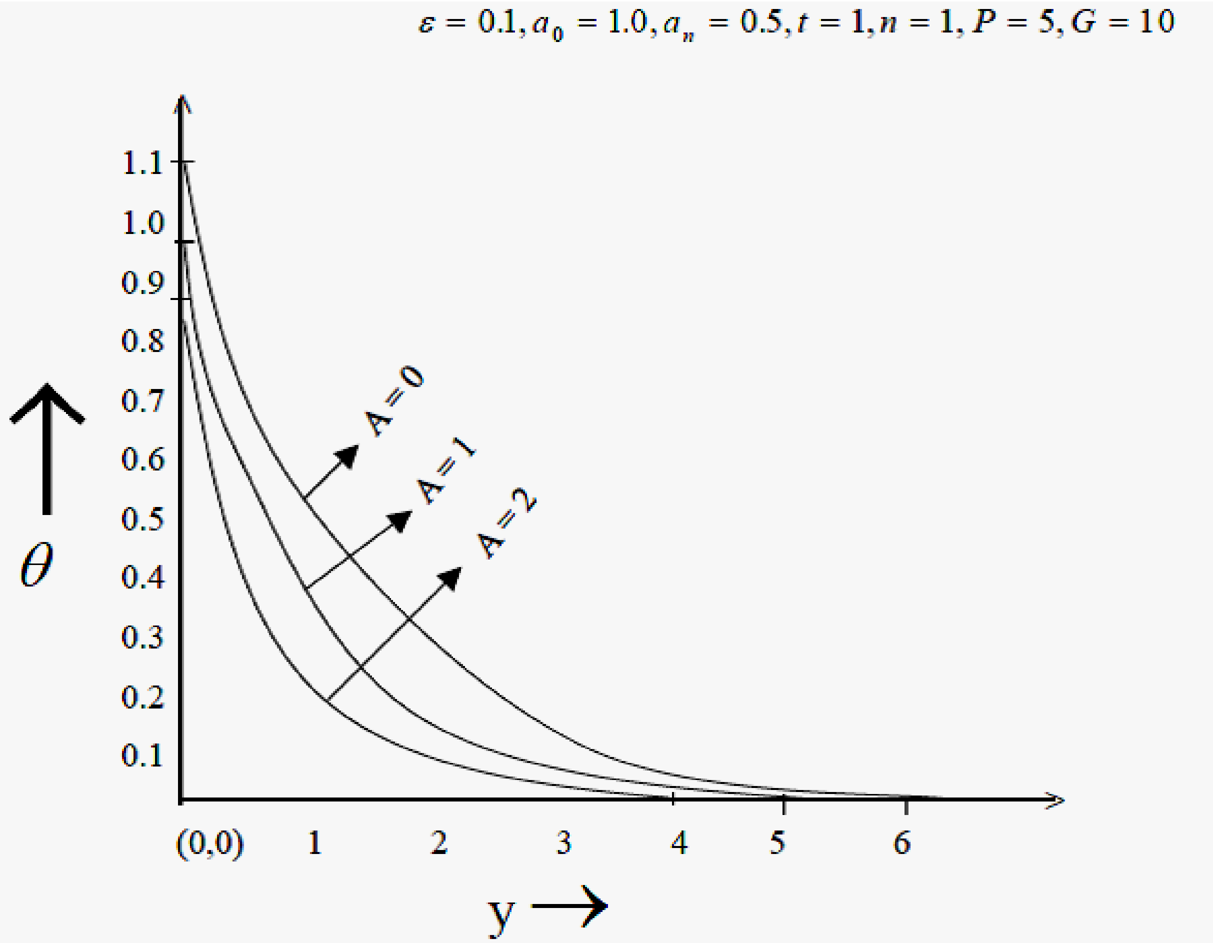

Fig. 3 : Exponentially increasing case : Temperature θ against y

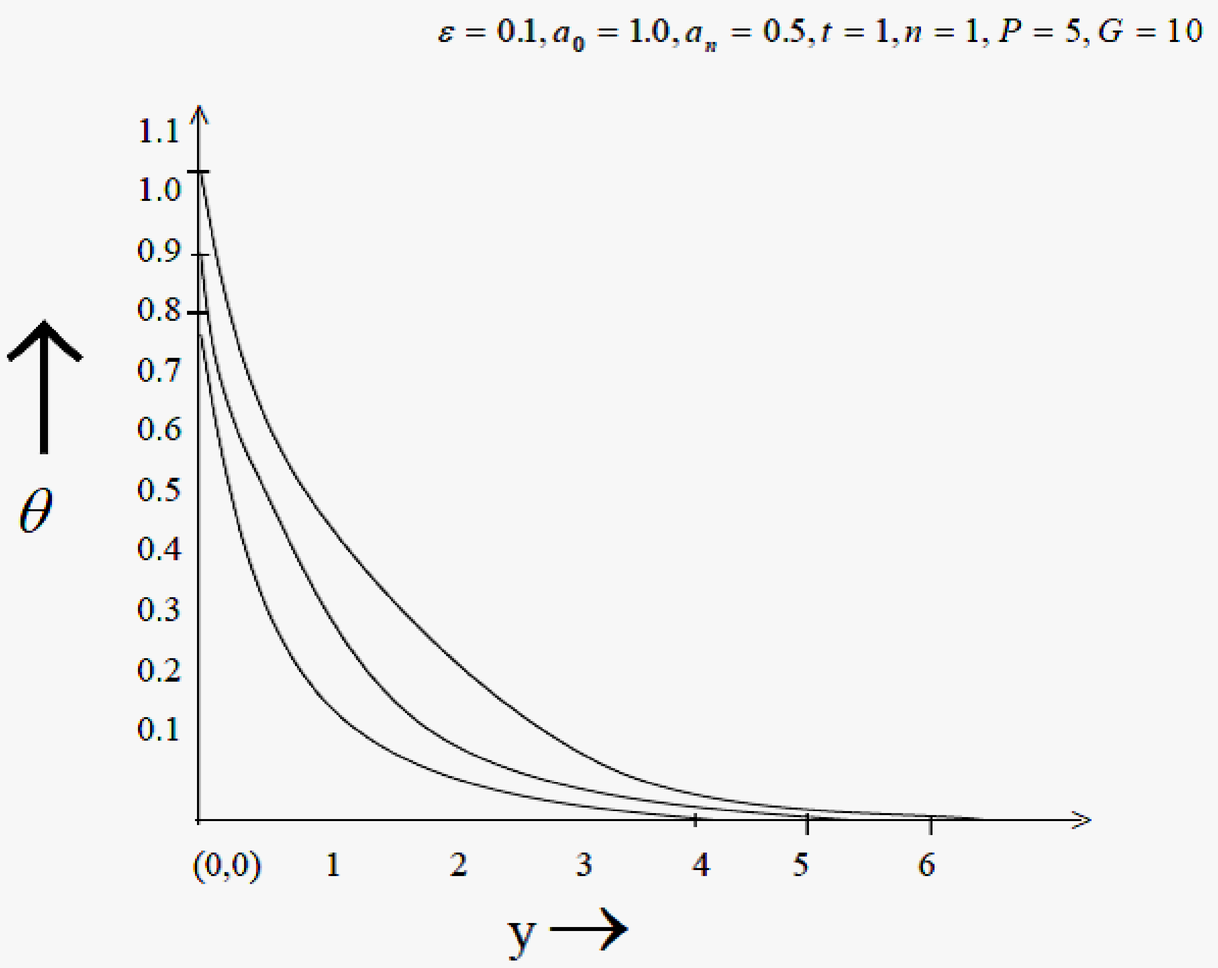

Fig. 4 : Exponentially decreasing case : Temperature θ against y

5. APPLICATIONS OF EXPONENTIAL FLOWS :

The study of exponential flow could be relevant to the treatment of cardiovascular diseases since cardiovascular flow in the human body often follows exponentially decreasing or increasing profiles. Exponential flow of Newtonian fluid was studied first, in order to arrive at an understanding of the proper mechanism, although blood flows itself shows some-Newtonian properties.

6. COMPARISON WITH PREVIOUS WORKS OF INVESTIGATORS :

The expression for velocity u given in (3.28) reduced to that of Kishore and Pandey (1972) for non-convective flow and the conclusions drawn from the graphs remain almost the same. The free convection effect was not considered earlier for exponential flow, though it has been considered by Soundalgeker (1972) for oscillating MHD flow past an infinite vertical late with variable suction. He has studied the effect of magnetic field on the velocity profile and concluded that the velocity field decreases with increase in magnetic parameter M. Thee shapes of his velocity profiles are entirely different from our graphs but conclusions are of similar nature.

Comparing our conclusions on temperature profiles with those of Mishra and Mohapatra (1975) we observe that the shapes of the temperature profiles are entirely different but the suction parameter A has similar effect on the temperature field. That is, a decrease in temperature profile is observed with increase in suction parameter A for both exponential and oscillatory flow.

7. REFERENCES

1. M.J. Lighthill, Proc. R. Soc. (London) A, 224 (1954), pp. 1-23.
2. J.T. Stuart, Proc. R. Soc. (London) A, 231 (1955), pp. 116-130.
3. S.A.S. Messiha, Proc Camb. Phil. Soc. 62 (1966), p. 329.
4. V.M. Soundelgaker, Indian J. Pure and appl. Math. 3 (3) (1972) pp. 426-436.
5. V.M. Soundalgeker, Bull. cal. Math. Soc. 68 (1976) pp. 301-313.
6. K.S. Pandey, Ind. J. phys. 12(4) (1968), pp. 213-222.
7. K.S. Pandey, J.Ind. Math. Soc 35(1971), pp. 87-98.
8. N. Kishore and R.D. Pandey, (1972), Ph.D. thesis B.H.U. Varanasi.
9. S. Mishra and P. Mohapatra. Indian J. Pure and appl. Math, 6(2) (1975) p.90.

****