MULTI-OBJECTIVE NON-LINEAR RELIABILITY GLOBAL OPTIMIZATION MODEL OF LIQUID CRYSTAL DISPLAY (LCD) USING FERMATEAN FUZZY OPTIMIZATION TECHNIQUES.

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Abstract: In this paper, we propose a multi-objective non-linear reliability global optimization and system cost as two objective functions. As a generalized version of fuzzy set, intuitionistic fuzzy set, pythagorean fuzzy set, fermatean fuzzy set is a very useful tool to express uncertainty, impreciseness in more general way. we have considered fermatean optimization technique with linear and non-linear membership function to solve this multi-objective reliability optimization model. To demonstrate the methodology and applicability of the proposed approach, numerical examples are presented and evaluated by comparing the result obtained by fermatean approach with the intuitionistic fuzzy optimization technique at the end of the paper.

Key words: Fuzzy set, Intuitionistic fuzzy set, Pythagorean fuzzy set, Fermatean fuzzy set, Reliability multi-objective programming, optimization, LCD, non- linear, upper bound.

1.Introduction:

In 1965, Zadeh [18] first introduced the concept of fuzzy set. The fuzzy settheory which considers the degree of membership of elements, is a very effective tool to measure uncertainty in real life situation. In recent time, the fuzzy settheory has been widely developed and generalized form have appeared. Intuitionistic fuzzy set (IFS) theory is one of the generalized versions of fuzzy set theory. In 1986, Atanassov [1] extended the concept of fuzzy set and introduced intuitionistic fuzzy set theory, which consider not only the degree of membership but also the degree of non-membership function such that the sum of both values is less than one. As a generalization of fuzzy set theory [18], intuitionistic fuzzy set theory [1], interval valued fuzzy sets [18,19] etc.Reliability engineering is one of the important tasks in designing and development of a technical system. The primary goal of the reliability engineer has been always to find the best way to increase system reliability. The diversity of system resources, resource constraints and options for reliability improvement lead to the construction and analysis of several optimization models. In daily life, due to some uncertainty in judgements of the decision maker (DM), there are some coefficients and parameters in the optimization model, which are alwaysimprecise with vague in nature. In order to handle such type of nature in multiobjective optimization model, fuzzy approach is used to evaluate this. Park [8] firstapplied fuzzy optimization techniques to the problem of reliability apportionment for a series system. Ravi et al. [9] used fuzzy global optimization reliability model. Huang [2] presented a multi objective fuzzy optimization method to reliability optimization problem. Later, intuitionistic fuzzy optimization method is also applied to various field of research work. Sharma [13] proposed a method to analyse the network system reliability which is based on intuitionistic fuzzy set theory. Jana and Roy [3] described intuitionistic fuzzy linear programming method in transportation problems. Also, Mahapatra [4] introduced intuitionistic fuzzy multi objective mathematical programming on reliability optimization model. Nowadays neutrosophic optimization technique is an open field of research work..In this paper, we propose a multi-objective non-linear reliability optimization and system cost as two objective functions. As a generalized version of fuzzy set, intuitionistic fuzzy set, pythagorean fuzzy set, fermatean fuzzy set is a very useful tool to express uncertainty, impreciseness in more general way. we have considered fermatean optimization technique with linear and non-linear membership function to solve this multi-objective reliability optimization model. To demonstrate the methodology and applicability of the proposed approach, numerical examples are presented and evaluated by comparing the result obtained by fermatean approach with the intuitionistic fuzzy optimization technique at the end of the paper.

Section:2 Preliminaries

Definition-2.1: A fuzzy set $\widetilde{\mathrm{D}}$ in X is a set of ordered pairs $\widetilde{\mathrm{D}} = \{(\mathrm{x}, \mu_{\widetilde{\mathrm{D}}}(\mathrm{x}))/\mathrm{x} \in \mathrm{X}\}$, where X is a collection of objects denoted generically by x and $\mu_{\widetilde{\mathrm{D}}}(\mathrm{x}): X \to [0,1]$ is called the membership function or grade of membership of x in $\widetilde{\mathrm{D}}$.

Example 2.2: Let $\mathrm{X} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$ be a set. Then the fuzzy set is given by

X	a	b	c	d
$\mu_{\widetilde{\mathrm{D}}}(\mathrm{x})$	0.3	0.2	0.5	0.7

Definition-2.3: Let Xbe a nonempty set. An intuitionistic fuzzy set Din Xis an object having the form $\mathrm{D} = \{\langle \mathrm{x}, \alpha_{\mathrm{D}}(\mathrm{x}), \beta_{\mathrm{D}}(\mathrm{x})\rangle/\mathrm{x} \in \mathrm{X}\}$, where the functions $\alpha_{\mathrm{D}}: \mathrm{X} \to [0,1]$ is the degree of membership and $\beta_D: X \to [0,1]$ is the degree of non – membership of $x \in X$ to D, with the condition $0 \le \alpha_D(x) + \beta_D(x) \le 1$.

Definition-2.4: A Pythagoreanfuzzy set D on a set X is defined by$D = \{(x, (\alpha_D(x), \beta_D(x))/x \in$ Xwhere αD:X→[0,1] is the degree of membership and βD:X→[0,1] is the degree of non – membership of $x \in X$, respectively which fulfil the condition $0 \le {\alpha_D}^2(x) + {\beta_D}^2(x) \le 1$ for all $x \in X$.

Definition 2.5:[Senapati and Yager, 2019a] Let 'X' be a universe of discourse A. Fermatean fuzzy set "F" in X is an object having the form $D = \{(x, (\alpha_D(x), \beta_D(x))/x \in X\}$, where $\alpha_D(x): X \to [0,1]$ and $\beta_D(x): X \to [0,1]$, including the condition $0 \le (\alpha_D(x))^3 + (\beta_D(x))^3 \le 1$, $for\ all\ x \in X$. The numbers $m_F(x)$ denotes the degree of membership and $n_F(x)$ denotes the non-membership of the element 'x' in the set F. Throughout this paper, we will denote a fermatean fuzzy set is FFS.

For any FFS 'F' and $x \in X$, $\pi_F(x) = \sqrt[4]{1-(\alpha_D(x))^2 - (\beta_D(x))^2}$ is identified as the degree of indeterminacy of 'x' to F. For convenience, Senapati and Yager [12]called $(\alpha_D(x),\ \beta_D(x))$ a fermatean fuzzy number (FFN) denoted by $F = (\alpha_D, \beta_D)$.

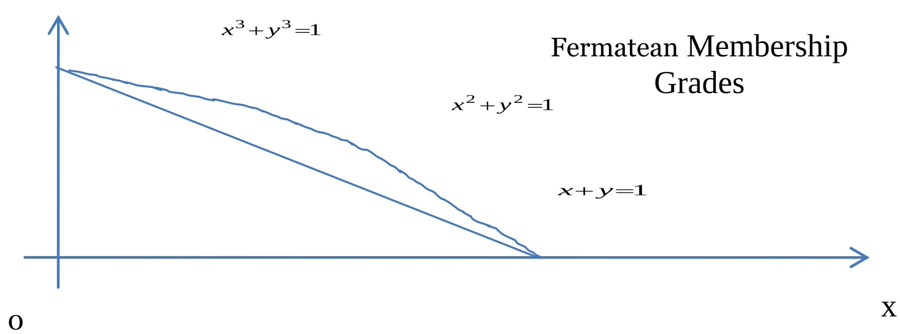

Fig-1

We shall point out the membership grades (MG's) related fermatean fuzzy sets as fermatean membership grades.

The various development of uncenrtainty model structures is defined in the following manner. This is the extension structures of uncertainties to fermatean fuzzy structures.

As a generalization of fuzzy set theory [18], intuitionistic fuzzy set theory [1], interval valued fuzzy sets [18,19] etc. Reliability engineering is one of the important tasks in designing and development of a technical system. The primary goal of the reliability engineer has been always to find the best way to increase system reliability. The diversity of system resources, resource constraints and options for reliability improvement lead to the construction and analysis of several optimization models.To demonstrate the methodology and applicability of the proposed approach, numerical examples are presented and evaluated by comparing the result obtained by fermatean approach with the intuitionistic fuzzy optimization technique at the end of the paper.

Fermatean fuzzy Development Model structure

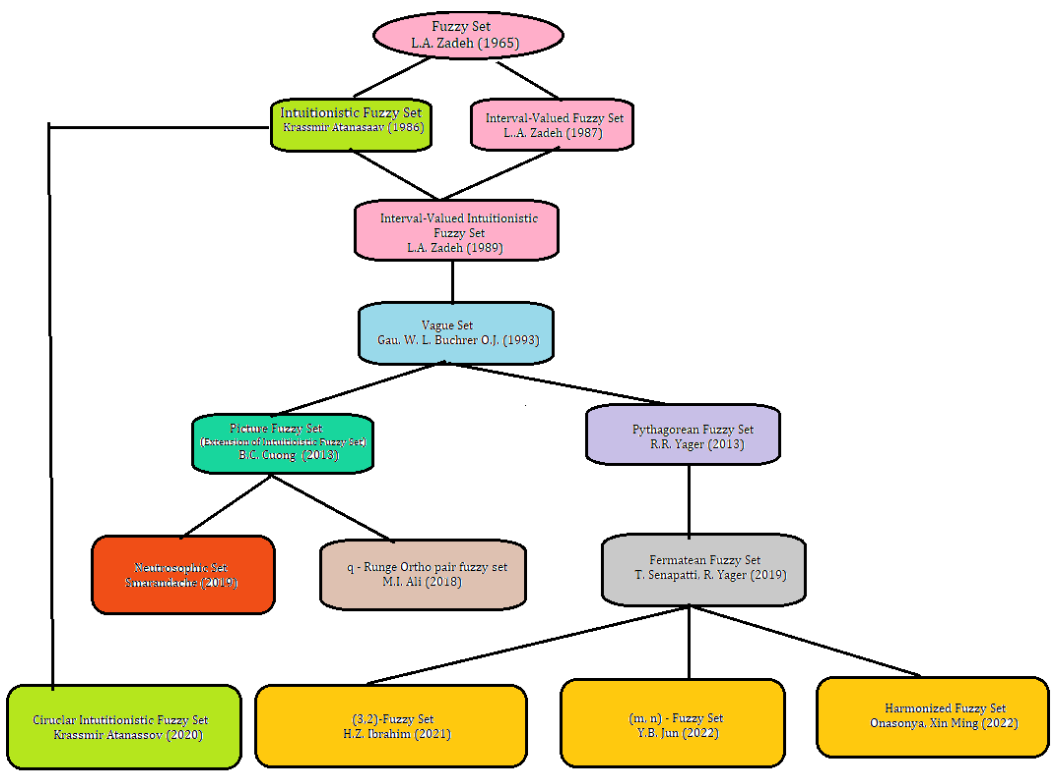

Fig-2

Section:3Mathematical Model:

Nowadays neutrosophic optimization technique is an open field of research work..In this paper, we propose a multi-objective non-linear reliability optimization and system cost as two objective functions. As a generalized version of fuzzy set, intuitionistic fuzzy set, pythagorean fuzzy set, fermatean fuzzy set is a very useful tool to express uncertainty, impreciseness in more general way. we have considered fermatean optimization technique with linear and non-linear membership function to solve this multi-objective reliability optimization model. To demonstrate the methodology and applicability of the proposed approach, numerical examples are presented and evaluated by comparing the result obtained by fermatean approach with the intuitionistic fuzzy optimization technique at the end of the paper.

Let E_j be the reliability of the j^{th} component of a system and $E_S(R)$ represents the system reliability. Let $D_S(R)$ denote the cost of the system. Here we consider a complex system, which includes a five –stage combination reliability model.

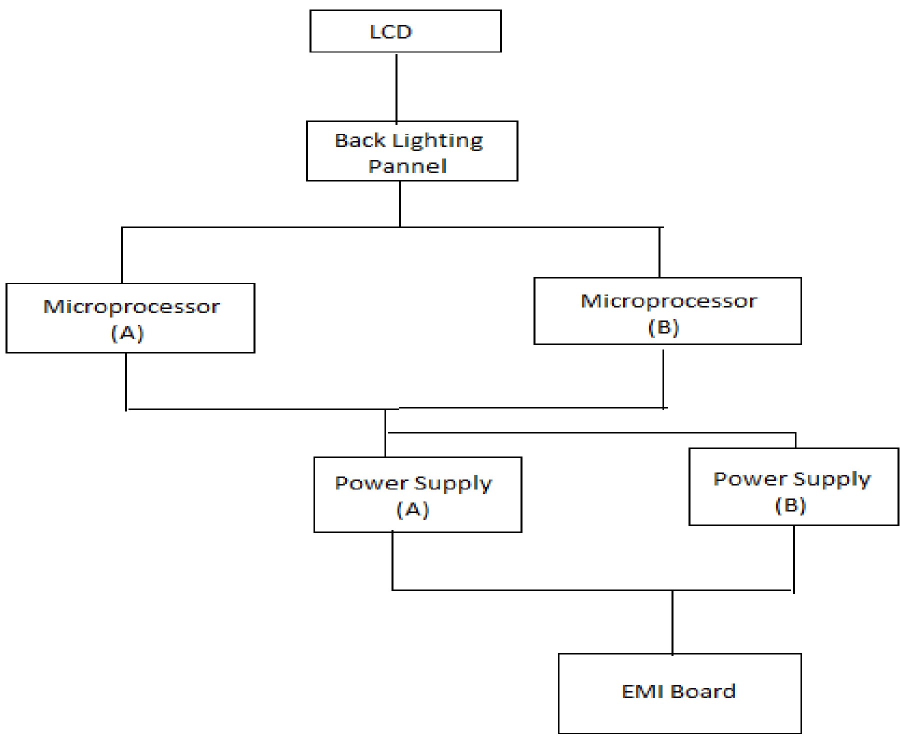

Fig-3

3.1. Reliability model of a LCD display Unit

Now we are interested to find out the system reliability of LCD is display unit which consists of several components connected to one another. This complex system mainly consists five stagesR_i(i=1,2,3,4,5) which are in series.

Thus, the generalized formula for the system reliability of the proposed model is given by

$E_S(R) = R_1 \times R_2 \times R_3 \times R_4 \times R_5 = \bigcap_{i=1}^{5} R_i$ ……………………(1)

where$R_1 = $ LCD panel with Reliability T_i ;

i.e. $R_1 = T_1$.

$R_2 = $ A black lighting board with 10 bulbs with individual bulb Reliability T_2

i.e.$R_2 = {T_2}^{10} + 10{T_2}^9(1 - T_2)$;

$R_3 = $ Two microprocessor boards A and B hooked up in parallel, each with reliability T_3.

i.e.$R_3 = 1 - (1 - T_3)^2$;

$R_4 = $ Dual power supplies in stand by redundancy, each power supply with reliability T_4.

i.e.$R_4 = T_4 + T_4 \log\left({}^{1}/_{T_4}\right)$;

$R_5 = $ EMI board with Reliability T_5 hooked in series with common input of the power supply A.

i.e. $R_5 = T_5$;

Thus, we have the following system reliability

$$E_S(R) = T_1\left({T_2}^{10} + 10{T_2}^9(1 - T_2)\right)(1 - (1 - T_3)^2)\left(T_4 + T_4 \log\left({}^{1}/_{T_4}\right)\right)T_5 \ldots(2)$$

3.2 Multi-Objective Reliability optimization method:

Here we consider cost of the proposed complex system as an additional objective function. Now system reliability has to be maximized and cost of the system is to be minimized subject to system space as target goal. Thus, the model becomes

$$\text{Max } C_S(R) = T_1\left({T_2}^{10} + 10{T_2}^9(1 - T_2)\right)(1 - (1 - T_3)^2)\left(T_4 + T_4 \log\left({}^{1}/_{T_4}\right)\right)T_5$$

$$\text{Min } D_S(R) = \sum_{j=1}^{5} d_j \left[\left(\tan\frac{\pi}{2}\right)T_j\right]^{r_j} \text{such that}$$

$$V_S(R) = \sum_{j=1}^{5} V_j {T_j}^{r_j} \leq V_{lim}$$

$0.5 \leq T_{j,min} \leq T_j \leq 1, 0 \leq T_S \leq 1$; $j = 1,2,3,4,5$…….(3)

where V_j and d_j represent the space and cost of the j^{th}component of the system respectively. V_{lim} is the system space limitation and $T_{j,min}$ is the lower bound of the reliability of each component j.

Now for simplicity of calculation and to convert the above problem to one type maximization problem, we consider $D'_S(R) = -D_S(R)$.

Thus the model (3) having the following form Max $E_S(R)$ Max $D'_S(R)$……….(4)

Subject to the same constraints defined in (3).

3.3 Mathematical Analysis Computational Algorithm

Here we are presenting a computational algorithm to solve multi objective reliability optimization model by simple values fermatean optimization approach another following steps are used.

Step-1: A multi-objective non-linear programming taking k objective functions can be taken asMaximize $(\varphi_1(x), \varphi_2(x), \ldots \varphi_k(x))$

subject to $f_i(x) \leq b_i$, i=1,2,…,m, $x \geq 0$……………………………………………..(1)

Step-2: Solve the above multi-objective non-linear programming model (1) taking one objective function at a time and avoid the others, so that we can get the ideal solutions. With the values of all objective functions evaluated at these ideal solutions, the pay-off matrix can be formulated as follows.

	φ_1	φ_2	…………….	φ_k
x_1	$\varphi_1{}^*(x_1)$	$\varphi_2(x_1)$	………………	$\varphi_k(x_1)$
x_2	$\varphi_1(x_2)$	$\varphi_2{}^*(x_2)$	………………	$\varphi_k(x_2)$
.				
.				
.				
x_k	$\varphi_1(x_k)$	$\varphi_2(x_k)$	………………	$\varphi_k{}^*(x_k)$

Table-1

Table-1 pay-off matrix of the solution of k single objective non-linear programming problem.

Step-3: Determine the upper bound and lower bound for each objective functions as follows.

$U_r^T = \max\{\varphi_r(x_1),\ \varphi_r(x_2), \ldots\ldots\ldots\ldots,\ \ \varphi_r(x_k)\}$, for all r=1, 2, ……, k and

$L_r^T = \min\{\varphi_r(x_1),\ \varphi_r(x_2), \ldots\ldots\ldots\ldots,\ \ \varphi_r(x_k)\}$, for all r=1, 2, ……, k …………(2)

So$L_r^T \leq \varphi_r(x) \leq U_r^T$. Here L_r^T andU_r^T are respectively lower and upper bounds of membership functions (objectives) $\varphi_r(x)$, for all r.

<u>Step-4</u>: Now the upper and lower bounds for objectives can be presented as follows.

$U_r^F = U_r^T - t_2(U_r^T - L_r^T)$ and

$L_r^F = L_r^T$ for all r…………………………………………………………(3)

Here L_r^F andU_r^F are respectively lower and upper bounds of membership of the r-th objective function$\varphi_r(x)$.

Hence t_2 lies between 0 and 1.

<u>Step-5</u>: Construct the truth membership and false membership functions as follows.

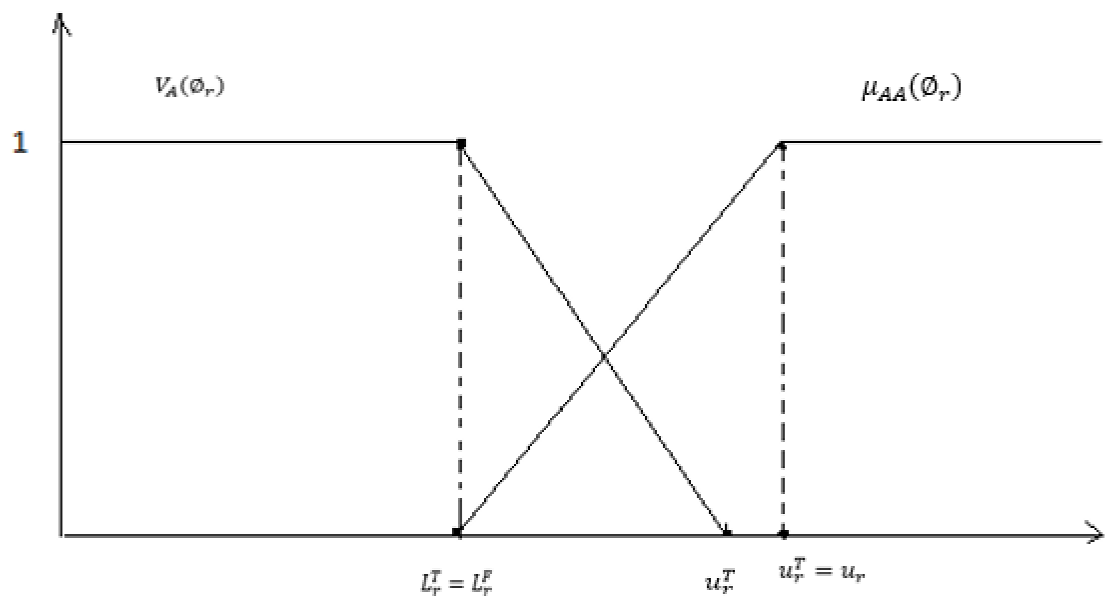

Fig-4

(a)Linear membership function:

$$\mu_A(\varphi_r) = \begin{cases} 0, & \varphi_r(x) \le L_r^{T} \\ \frac{\varphi_r(x) - L_r^{T}}{U_r^{T} - L_r^{T}}, & L_r^{T} \le \varphi_r(x) \le U_r^{T}, r = 1, 2, \ldots \ldots, k \\ 1, & \varphi_r(x) \ge L_r^{T} \end{cases} \ldots\ldots\ldots\ldots(4)$$

and

$$\gamma_A(\varphi_r) = \begin{cases} 0, & \varphi_r(x) \le L_r^{F} \\ \frac{U_r^{F} - \varphi_r(x)}{U_r^{F}}, & L_r^{F} \le \varphi_r(x) \le U_r^{F}, r = 1, 2, \ldots \ldots, k \\ 1, & \varphi_r(x) \ge U_r^{T} \end{cases} \ldots\ldots\ldots\ldots(5)$$

(a) Non-Linear membership function:

$$\mu_A(\varphi_r) = \begin{cases} 0, & \varphi_r(x) \le L_r^{T} \\ 1 - \exp\left(-\chi \frac{\varphi_r(x) - L_r^{T}}{U_r^{T} - L_r^{T}}\right), & L_r^{T} \le \varphi_r(x) \le U_r^{T}, r = 1, 2, \ldots \ldots, k \\ 1, & \varphi_r(x) \ge L_r^{T} \end{cases} \ldots\ldots(6)$$

and

$$\gamma_A(\varphi_r) = \begin{cases} 1, & \varphi_r(x) \le L_r^{F} \\ \frac{1}{2} + \frac{1}{2} tanh\left(\Delta_r \left(\frac{U_r^{F}}{2}\right) - \varphi_r(x)\right), & L_r^{F} \le \varphi_r(x) \le U_r^{F}, r = 1, 2, \ldots \ldots, k \\ 0, & \varphi_r(x) \ge U_r^{T} \end{cases} \ldots\ldots(7)$$

where χ and Δ_r are two non-zero parameters prescribed by the decision maker.

Step-6: Now, using fermatean optimization technique the given multi-objective non-linear programming (MONLP) is equivalent to the following non-linear problem as

Max $\mu_r(\varphi_r(x))$

Min $\gamma_r(\varphi_r(x))$

subject to $\mu_r(\varphi_r) \ge \gamma_r(\varphi_r)$

$\gamma_r(\varphi_r) \ge 0$

$0 \le \mu_r(\varphi_r) + \gamma_r(\varphi_r) \le 1$

$h_i(x) \le b_i$, i=1, 2, …..,m, x ≥ 0, for all r = 1, 2, 3, …., k………..(8)

where $\mu_r(\varphi_r)$ and $\gamma_r(\varphi_r)$ the membership functions of fermatean decision set respectively.

Step-7: Now using additive operator, the above equation (8) reduced to the following crisp model.

Maximize

$$\sum_{r=1}^{k}\left(\mu_r(\varphi_r)-\gamma_r(\varphi_r)\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Subject to the same constraints described in (8) and (9).

Step-8: Solve equation (9) to get optimal solution.

Section:4 Fermatean optimization technique on multi-objective reliability optimization problem.

To solve the above defined problem in Table-1, pay-off matrix is formulated as follows.

	$E_S(R)$	$D_S'(R)$
T_1	$E_S{}^*(T_1)$	$D_S'(T_1)$
T_2	$E_S(T_2)$	$D_S'(T_2)$

Now the best upper bound and worst lower bound are identified. The upper and lower bound for the membership functions of the objective functions are define as

$$U_{E_S}{}^{T} = \max\{E_S(R_1), E_S(R_2)\}$$

$$U_{D_S'}{}^{T} = \max\{D_S'(R_1), D_S'(R_2)\}$$

$$L_{E_S}{}^{T} = \min\{E_S(R_1), E_S(R_2)\}$$

$$L_{D_S'}{}^{T} = \min\{D_S'(R_1), D_S'(R_2)\}$$

……………………………(10)

where $L_{E_S}{}^{T} \le E_S(R) \le U_{E_S}{}^{T}$ and

$L_{D_S'}{}^{T} \le D_S'(R) \le U_{D_S'}{}^{T}$.

Also the upper and lower bound for falsity membership objective functions can be presented as

$$U_{E_S}{}^{F} = U_{E_S}{}^{T} - t_2\left(U_{E_S}{}^{T} - L_{E_S}{}^{T}\right) \text{ and}$$

$L_{E_S}{}^{F} = L_{E_S}{}^{T}$

$U_{D_S{}'}{}^{F} = U_{D_S{}'}{}^{T} - t_2\left(U_{D_S{}'}{}^{T} - L_{D_S{}'}{}^{T}\right)$ and

$L_{D_S{}'}{}^{F} = L_{D_S{}'}{}^{T}$

…………………………………………………….(11)

Now the linear and non-linear membership functions are formulated for objective functions E_S (R) and $D_S{}'(R)$. After electing the membership functions, the crisp non-linear programming problem is formulated as follows.

Maximize $\{\mu_{E_S}(E_S(R)) + \mu_{D_S{}'}(D_S{}'(R)) - \gamma_{E_S}(E_S(R)) - \gamma_{D_S{}'}(D_S{}'(R))\}$

subject to

$\mu_{E_S}(E_S) \geq \gamma_{E_S}(E_S)$,

$\mu_{D_S{}'}(E_S) \geq \gamma_{D_S{}'}(E_S)$,

$0 \leq \mu_{E_S}(E_S) + \gamma_{E_S}(E_S) \leq 1$ and

$0 \leq \mu_{D_S{}'}(E_S) + \gamma_{D_S{}'}(D_S{}') \leq 1$.

And $\gamma_{E_S}(E_S) \geq 0$, $\gamma_{D_S{}'}(D_S{}') \geq 0$………………………………..………..(12)

$$\sum_{j=1}^{5} V_j\ E_j{}^{a_j} \leq V_{lim}$$

$$0.5 \leq E_{j,min} \leq E_j \leq 1,$$

$0 \leq E_S \leq 1$, $j = 1,2,3,4,5$.

Solve the above crisp model to obtain optimal solution of the system reliability and cost of the system.

4.1.Numerical Example:

In this numerical example, we have a five stage combination reliability model of a complex system is considered for numerical exposure.

The problem becomes as follows,

Max $E_S(R) = T_1\left(T_2{}^{10} + 10T_2{}^{9}(1-T_2)\right)(1-(1-T_3)^2)\left(T_4 + T_4\log\left({}^{1}/_{T_4}\right)\right)T_5$

Min $E_S(R) = \sum_{j=1}^{5} C_j\left(\left(\tan\frac{\pi}{2}\right)T_j\right)^{\alpha_j}$

Subject to constraints

$\gamma_S(R) = \sum_{j=1}^{5} V_j\ T_j{}^{a_j} \leq V_{lim}$

$$0.5 \leq T_{j,min} \leq T_j \leq 1,$$

$0 \leq E_S \leq 1$, $j = 1,2,3,4,5$……………(1)

Table-1The input data is given below.

C_1	C_2	C_3	C_4	C_5	V_1	V_2	V_3	V_4	V_5	$\alpha_j (\forall j)$	$a_j (\forall j)$	V_{lim}
9	28	34	37	31	4	3.75	1	2	6	0.75	1	20

Table-2 With the above solutions pay-off-matrix of the objective functions is formulated as follows.

	E_S (R)	D_S ′(R)
T_1	0.8317	−5324.21
T_2	0.0012	−123.32

Now the upper and lower bound for the truth membership of objective functions are given by

$U_{E_S}{}^T = 0.8317$

$U_{D_S{}'}{}^T = -123.32$

$L_{E_S}{}^T = 0.0012$

$L_{D_S{}'}{}^T = -5324.21$ and can be written as

$0.0012 \leq E_S(R) \leq 0.8317$

$-5324.21 \leq D_S{}'(R) \leq -123.32.$

The upper and lower bound for the other membership functions of objective functions can be presented as

$U_{E_S}{}^F = 0.8317 - t_2(0.82210)$and

$L_{E_S}{}^F = 0.0012$

$U_{D_S{}'}{}^F = -123.32 - t_2(0.82210)$and

$L_{D_S{}'}{}^F = -5324.21$

Here we consider $t_1 = 0.002$, $t_2 = 0.085$. Also $\Delta_1 = 1.12$, $\Delta_2 = 0.001$ and χ=3.

Table-3 Comparison of optimal solutions by intuitionistic fuzzy optimal, Pythagorean fuzzy optimal and fermatean fuzzy optimal technique.

Method	T_1	T_2	T_3	T_4	T_5	E_S (R)	$D_S{}'$(R)
Intuitionistic fuzzy optimal	0.8321	0.8431	0.7601	0.7321	0.8014	0.7011	724.31
Pythagorean fuzzy optimal	0.8323	0.8434	0.7610	0.7324	0.8021	0.7123	680.23
Fermatean fuzzy optimal (LMF)	0.8337	0.8534	0.7821	0.7335	0.8041	0.7130	631.2
Fermatean fuzzy optimal (NLMF)	0.8341	0.8536	0.7831	0.7341	0.8045	0.7141	732.5

The above table shows the comparison of results of the proposed approach with the intuitionistic fuzzy optimal approach and Pythagorean fuzzy optimal approach. It is clear from the table-3that fermatean fuzzy technique with linear membership function gives more of less same system reliability that the system cost with Intuitionistic fuzzy optimization, Pythagorean fuzzy optimization.

However, in perspective of system reliability Fermatean fuzzy optimization technique with non-linear membership function gives better result than Intuitionistic and Pythagorean approach.

Conclusion: Here we have introduced Fermatean fuzzy optimization technique with linear and non-linear membership function to find the optimal solution of the proposed multi-objective non-linear reliability global optimization model. The main purpose of this paper is to give a computational procedure for solving multi-objective reliability optimization model by fermatean optimization technique to find the optimal solution, which maximize the system reliability and minimize the cost of the system.We have obtained the result in the Fermatean optimization technique was compared with the intuitionistic fuzzy optimization and Pythagorean fuzzy optimization and it shows that the fermatean fuzzy optimization with non-linear membership function gives better reliable system. Thus, the proposed method is an efficient and modified optimization techniques and gives a highly reliable system than the other existing method.

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