SINGLE -VALUED PYTHAGOREAN FUZZY NET PRESENT WORTH ECONOMIC ANALYSIS UNDERCAPITAL BUDGETING TECHNIQUES

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Abstract: Capital budgeting techniques are used to determine an equivalent value for each flows of an investment project to decide whether it is profitable or not. In this paper, present worth analysis is done using single valued Pythagorean fuzzy sets to increase the uncertainty taken into account in the economic analysis. Also fuzzy capital budgeting techniques that are fuzzy net present worth, fuzzy net future worth and fuzzy net annual worth are extended using Pythagorean fuzzy sets. An illustration for the calculation is also presented.

Keywords: Fuzzy set, Pythagorean fuzzy set, Capital budget, Single-valued Pythagorean, worth analysis, Net present work, Annual worth.

1.Introduction: Uncertainties about the future business to make a plan and take quick decisions. In today, global economics contain high uncertainty which forces business that are one of the elements of the economic system to make their investment decision under high uncertain conditions to survive again competitors[3]. Under increasing and lethal competition conditions, the sustainability of business depends on making a profit, and in other to achieve this goal, it is necessary to make connect and reliable decisions on investments. Effective and efficient implementations of the decisions are taken as a management issue too. Within this economic system, the responsibilities of decision-makers can be quite high order to carry out actions that will ensure companies survival, such as keeping up with the developing technology, forming the right strategies, and outperforming the competition. In an environment where rapid fluctuations and competition are experienced, for the companies whole first and main goal is to make profit, it is possible to achieve this goal with proper investment planning and the harmony and coordination of the factors that determine the profit [4].

Using fuzzy logic theory in investment analysis enables decision makers to approximately evaluate the investments planned by the companies before operating activities are concluded and to estimate their financial returns especially in fuzzy economics. The profitability of business that are difficult to predict includes uncertainty due to fluctuations in global economics. Thus, business have the opportunity to make plans for the future.Pythagorean fuzzy sets are one of the most recent extensions of fuzzy set theory which are capable to handle higher level of uncertainties by assigning future parameters from a larger domain.

In this article, we analyze, thepresent worth analysis is done using single valued Pythagorean fuzzy sets to increase the uncertainty taken into account in the economic analysis. Also fuzzy capital budgeting techniques that are fuzzy net present worth, fuzzy net future worth and fuzzy net annual worth are extended using Pythagorean fuzzy sets. An illustration for the calculation is also presented.The rest of the paper is organized as follows:

In section-2, a detailed literature review of fuzzy capital budgeting techniques is given. The preliminaries of various Pythagorean fuzzy sets are determined in section-3. In section-4, Pythagorean fuzzy net present worth formulas are proposed and applied on an illustrative example. The paper lasts with conclusion and future research suggestions.

2. Literature Survey: Chiu and Park[6] Cultivated the fuzzy present value analysis by expressing fuzzy interest rates and fuzzy cash flows in their studies by triangular fuzzy numbers. When the literature on fuzzy capital budgeting in analyzed, many academic studies have been found. Kahraman et.al[8] used the fuzzy benefit cost ratio method which is based on the new fuzzy budgeting technique.

Wang and Liang[16] proposed two algorithms that give the fuzzy benefit cost ratio and the fuzzy benefit cost increase rate in order to perform benefit cost analysis in fuzzy environment. Buckley[5] developed a fuzzy equivalent for compound and simple interest rate problems used in mathematics of finance and formulated the fuzzy present value and fuzzy future value using this interest rate. Pohjola and Turunan[ ] expanded the profitability analysis for fuzzy data by expressing the internal rate of return with type-2 fuzzy number.

Kuchta[] defined fuzzy payback time, fuzzy net present value, fuzzy net future, and fuzzy internal rate of return formulas and examined fuzzy cash flows, fuzzy interest rate and fuzzy project life by evaluating investments. Kavsak and Tolga[ ]used fuzzy discounted cash-flow analysis in their fuzzy decision algorithm to evaluate economic criteria.Kahraman et.al[13] developed the fuzzy net present value, fuzzy equivalent uniform annual value, fuzzy future value, fuzzy benefit-cost ratio and fuzzy payback period formulas for geometric and trigonometric cash flows in discrete and continuous discounting situation.

Since several fuzzy extensions of capital budgeting methods, have been proposed, Kahraman et.al[8] propose fuzzy net present value-based fuzzy engineering economy decision models in which uncertain cash flow and interest rate are expressed in triangular fuzzy numbers. Sari and Kahraman et.al[14] used both triangular and trapezoidal internal type-2 fuzzy sets to develop internal type-2 fuzzy capital budgeting techniques which are internal type-2 fuzzy net present value analysis. Kahraman et.al[12] developed interval-valued intuitionistic fuzzy present worth analysis for the evaluation of CNC(computer numerical control) lathe investments and more than one expert was evaluated through aggregation operators in line with these analysis. [Yager R.R 2013] introduced pythagorean fuzzy set.

Section:2 Preliminaries

Definition-2.1: A fuzzy set $\widetilde{D}$ in X is a set of ordered pairs $\widetilde{D} = \{(x, \mu_{\widetilde{D}}(x))/x \in X\}$, where X is a collection of objects denoted generically by x and $\mu_{\widetilde{D}}(x): X \to [0,1]$ is called the membership function or grade of membership of x in $\widetilde{D}$.

Example 2.2: Let $X = \{a, b, c, d\}$ be a set. Then the fuzzy set is given by

X	a	b	c	d
$\mu_{\widetilde{D}}(x)$	0.3	0.2	0.5	0.7

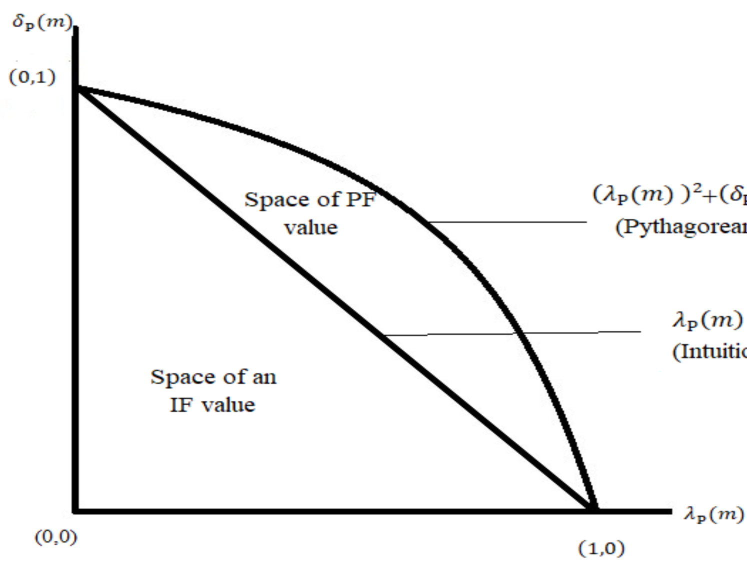
Definition-2.3: Let X be a nonempty set. An intuitionistic fuzzy set D in X is an object having the form $D = \{\langle x, \alpha_D(x), \beta_D(x)\rangle/x \in X\}$, where the functions $\alpha_D: X \to [0,1]$ is the degree of membership and $\beta_D: X \to [0,1]$ is the degree of non – membership of $x \in X$ to D, with the condition $0 \leq \alpha_D(x) + \beta_D(x) \leq 1$.

Definition-2.4: A Pythagorean fuzzy set D on a set X is defined by$D = \{(x, (\alpha_D(x), \beta_D(x))/x \in$ Xwhere αD:X→[0,1] is the degree of membership and βD:X→[0,1] is the degree of non – membership of $x \in X$, respectively which fulfil the condition $0 \leq {\alpha_D}^2(x) + {\beta_D}^2(x) \leq 1$ for all $x \in X$.

3. Pythagorean fuzzy sets and its Ranking function

Definition 3.1 Pythagorean fuzzy set : Let M be a fixed set. Then a pythagorean fuzzy set in M can be defined as follows: $P = \{(m, \lambda_P(m),\ \delta_P(m)) / m \in M\}$ ………………(1) where $\lambda_P(m)$ and $\delta_P(m)$ are mapping from M to [0, 1], with conditions $0 \leq \lambda_P(m) \leq 1, 0 \leq \delta_P(m) \leq 1$ and $0 \leq {\lambda_P}^2(m) + {\delta_P}^2(m) \leq 1$,………… (2) for all $m \in M$, and they denote the degree of membership and degree of non-membership of element $m \in M$ to the set

P, respectively. Let $\pi_P(m) = \sqrt{1 - {\lambda_P}^2(m) - {\delta_P}^2(m)}$, …………(3) then it is called the pythagorean fuzzy index of element $m \in M$ to set P; representing the degree of indeterminacy of m to P.Also $0 \leq \pi_P(m) \leq 1$ for every $m \in M$.



Aydin and Kabak[2] provided a new multiplication operator and a new term under the name of " neutrosophic equivalent" using present value and future value analysis techniques with single-valued neutrosophic set , proposed techniques were used to analysis and evaluate alternatives by experts.

Definition 3.2 Operations on Pythagorean fuzzy set: Let $A = (\mu_A,\ \gamma_A)$, $B = (\mu_B,\ \gamma_B)$ and $C = (\mu_C,\ \gamma_C)$ be the pythagorean fuzzy numbers, and α is a scalar that is greater than 0. The arithmetic operations are given in equations (4) to (10).

1. Summation: $B \oplus C = \left(\sqrt{{\mu_B}^2 + {\mu_C}^2 - {\mu_B}^2{\mu_C}^2},\ \gamma_B\gamma_C\right)$……………………..(4)
2. Subtraction:$B \ominus C = \left(\sqrt{\frac{{\mu_B}^2-{\mu_C}^2}{1-{\mu_C}^2}}\ ,\frac{\gamma_B}{\gamma_C}\right)$ if $\mu_B \geq \mu_C$, $\gamma_B \leq \min\left\{\gamma_C\ ,\frac{\gamma_C\pi_1}{\pi_2}\right\}$..(5)
3. Multiplication:$B \otimes C = \left(\mu_B\mu_c\ ,\ \sqrt[2]{{\gamma_B}^2 + {\gamma_C}^2 - {\gamma_B}^2{\gamma_C}^2}\right)$…………………..(6)
4. Scalar product:$\alpha A = \left(\sqrt{1-(1-\mu_A)^\alpha}, {\gamma_A}^\alpha\right)$…………………………….(7)
5. Exponentiation:$A^\alpha = \left({\mu_A}^\alpha, \sqrt[2]{1-(1-{\gamma_A}^2)^\alpha}\right)$……………………………(8)
6. Division: $B \div C = \left(\frac{\mu_B}{\mu_C}\ ,\ \sqrt[2]{\frac{{\gamma_B}^2-{\gamma_C}^2}{1-{\gamma_C}^2}}\right)$ if $\gamma_B \leq \min\left\{\mu_C\ ,\frac{\mu_C\pi_1}{\pi_2}\right\}$, $\gamma_B \geq \gamma_C$……(9)

Definition 3.2 Ranking of Pythagorean fuzzy sets: Yager[17] proposed the score functions and the accuracy function to provide a comparison between pythagorean fuzzy sets.

The score functions of $P = (\mu_P,\ \gamma_P)$, can be represented as follows for any pythagorean fuzzy number.

$\mathrm{score}(P) = {\mu_P}^2 - {\gamma_P}^2$ where the $\mathrm{score}(P) \in [-1, 1]$………………………………(10)

The accuracy functions of $P = (\mu_P,\ \gamma_P)$, can be described as follows for any pythagorean fuzzy number.

$\mathrm{acc}(P) = {\mu_P}^2 + {\gamma_P}^2$ where the $\mathrm{acc}(P) \in [0, 1]$……………………………………..(11)

The ranking techniques for any two pythagorean fuzzy numbers can be defined as follows Depending on the score and accuracy functions of pythagorean fuzzy numbers:

Let $P_1 = (\mu_{P_1},\ \gamma_{P_1})$ and $P_2 = (\mu_{P_2},\ \gamma_{P_2})$ be two pythagorean fuzzy numbers. $\mathrm{score}(P_1)$ and $\mathrm{score}(P_2)$, $\mathrm{acc}(P_1)$ and $\mathrm{acc}(P_2)$ are the score values and accuracy values of P_1 and P_2 .Then

$\mathrm{score}(P_1) < \mathrm{score}(P_2)$, then $P_1 < P_2$…………………………………………………..(12)

$\mathrm{score}(P_1) > \mathrm{score}(P_2)$, then $P_1 > P_2$…………………………………………………..(13)

$\mathrm{acc}(P_1) < \mathrm{acc}(P_2)$, then $P_1 < P_2$

$\mathrm{score}(P_1) \equiv \mathrm{score}(P_2)$, then

$\mathrm{acc}(P_1) > \mathrm{acc}(P_2)$, $P_1 > P_2$

$\mathrm{acc}(P_1) = \mathrm{acc}(P_2)$, $P_1 = P_2$………………………………………………………(14)

4.Pythagorean fuzzy net present worth analysis

Net present worth is one of the most used discounted cash flow methods in the evaluation of investment projects that can be described as the summation of the equivalent values of all cash flow of project on present time. The equivalent values of cash flows which are occurring on different time periods is calculated by interest formulas that represents the time effect on the cash flow amount. There are several assumptions that are made in the net present worth analysis. One of the critical assumptions is that the future-flows, future interest rates and other factors such as useful lives and timing of the cash receipts can be forecasted with certainty [Sullivan.M.G et.al]

Most of the future forecasts involve high uncertainty due to the unpredictable manner of the factors affecting the forecasted parameter. For this reason, as it is summarized in section-2, fuzzy extensions of net present worth analysis are proposed by various authors for different uncertainty levels. Fuzzy net present worth (FNPW) can be calculated using equation (15) where FNCF represents fuzzy net cash flow on timej, i represents fuzzy interest rate and n represents study period of the project:

$$\mathrm{FNPW}=\sum_{j=0}^{n}\mathrm{FNCF}_j\,(1+i)^{-n} \quad \ldots\ldots(15)$$

If the cash flows of an investment project have annual uniform values then FNPW can be formulated by equation (16), where I denotes initial investment cost which occurs at the beginning of the project, A denotes annual uniform net cash flow starting at the end of first period and lasts at the n^{th}period, and salvage value or residual value (SV) of the project.

$$\mathrm{FNPW}=-\mathrm{I}+\mathrm{A}((1+i)^{n}-1)(i(1+i)^{n})+\mathrm{SV}(1+i)^{-n} \quad \ldots\ldots(16)$$

Pythagorean fuzzy sets are capable to handle higher levels of uncertainties by assigning fuzzy parameters from a larger domain. Equation (17) is proposed to calculate pythagorean fuzzy net present worth (PFNPW) as follows:

$$\mathrm{PFNPW}(\mu,\gamma)=\sum_{j=0}^{n}\mathrm{FNCF}_j\,(\mu,\gamma)\big(1+i(\mu_i,\ \gamma_i)\big)^{-n} \quad \ldots\ldots(17)$$

when the parameters are determined using pythagorean fuzzy sets, equation (16) can be written as given in equation (18) as follows:

$$\mathrm{PFNPW}(\mu,\gamma)=-\mathrm{I}(\mu_I,\ \gamma_I)+A(\mu_A,\ \gamma_A).\frac{\big(1+i(\mu_i,\gamma_i)\big)^{-n}-1}{\big(i(\mu_i,\gamma_i)\big)\big(1+i(\mu_i,\gamma_i)\big)^{n}}+\frac{\mathrm{SV}(\mu_{SV},\gamma_{SV})}{\big(1+i(\mu_i,\gamma_i)\big)^{n}} \quad \ldots\ldots(18)$$

When the conditions of subtraction and division operations are satisfied, equation (18) can be rewritten as in equation (19) using the arithmetic operations that are given in equations (4) to (9).

$$PFNPW(\mu,\gamma) = \left(-I + \frac{A((1+i)^n-1)}{(i(1+i)^n)} + \frac{SV}{(1+i)^n}\right)(\mu,\gamma) \quad \ldots\ldots(19)$$

where $$\mu = \sqrt[2]{\frac{\frac{\mu_A{}^2}{\mu_i{}^2}+\frac{\mu_{SV}{}^2}{\mu_i{}^{3n}}-\frac{\mu_{SV}{}^2\mu_A{}^2}{\mu_i{}^{2n+2}}-\mu_I{}^2}{1-\mu_i{}^2}} \quad \ldots\ldots(20)$$

$$\gamma = \sqrt[2]{\frac{(1-\gamma_C{}^2)(\gamma_A{}^2-\gamma_i{}^2)+(\gamma_{SV}{}^2-1)(\gamma_A{}^2-\gamma_i{}^2)}{\gamma_I{}^2(1-\gamma_C{}^2)^{n+1}}} \quad \ldots\ldots(21)$$

If the conditions for subtraction and division operations are not satisfied in equation (19) then Equation (22) can be used to calculate PFNPW:

$$PFNPW(\mu,\gamma) = \left(-I + \frac{A((1+i)^n-1)}{(i(1+i)^n)} + \frac{SV}{(1+i)^n}\right)\min\left(\mu_A,\mu_{SV},\mu_i,\ \mu_I\right),\ \max\left(\gamma_A,\gamma_{SV},\gamma_i,\ \gamma_I\right) \quad \ldots\ldots(22)$$

5. An Illustration

Let project-I and project-II are two mutually exclusive investment projects which have the cash flows that are determined using pythagorean fuzzy sets as shown in Table-1. Pythagorean fuzzy interest rate is determined as 5% (0.6, 0.4) per year.

Cash flow of the project-I

	Project-I
Initial investment cost (I)	45,000 (0.2, 0.8)
Net annual income (A)	90,000 (0.3, 0.4)
Salvage value (SV)	15,000 (0.2, 0.3)
Interest value (i)	0.05 (0.6, 0.4)
Useful life (n)	5

Table-1

Equation (19) becomes

$$PFNPW(\mu,\gamma) = \left(-I + \frac{A((1+i)^n-1)}{(i(1+i)^n)} + \frac{SV}{(1+i)^n}\right)(\mu,\gamma)$$

$$= \left(-45{,}000 + \frac{90{,}000((1+0.05)^5-1)}{(0.05(1+0.05)^5)} + \frac{15{,}000}{(1+0.05)^5}\right)(0.6981, 0.6329)$$

$$= \$\ 4455.07\ (0.6981, 0.6329).$$

Using $\mu_A = 0.3$, $\mu_1 = 0.8$, $\mu_I = 0.2$, $\mu_{SV} = 0.2$, n=5 and

$\gamma_A = 0.4$, $\gamma_1 = 0.2$, $\gamma_I = 0.8$, $\gamma_{SV} = 0.3$, n=5 equation (20) becomes

$\mu = 0.6981$ and $\gamma = 0.6329$.

Also, Cash flow of the project-II

	Project-II
Initial investment cost (I)	55,000 (0.4, 0.7)
Net annual income (A)	1,10,000 (0.5, 0.4)
Salvage value (SV)	25,000 (0.4, 0.8)
Interest value (i)	0.05 (0.9, 0.3)
Useful life (n)	5

Table-2

Equation (19) becomes

$$\text{PFNPW}(\mu, \gamma) = \left(-55{,}000 + \frac{1{,}10{,}000\left((1+0.05)^5-1\right)}{\left(0.05(1+0.05)^5\right)} + \frac{25{,}000}{(1+0.05)^5}\right)(0.7446, 0.1808)$$

$= \$\ 5510.38\ (0.7446, 0.1808)$.

Using $\mu_A = 0.5$, $\mu_1 = 0.9$, $\mu_I = 0.4$, $\mu_{SV} = 0.4$, n=5 and

$\gamma_A = 0.4$, $\gamma_1 = 0.7$, $\gamma_I = 0.8$, $\gamma_{SV} = 0.3$, n=5 equation (20) becomes

$\mu = 0.7446$ and $\gamma = 0.1808$.

Using equation from (19) to (22),

PFNPW of project-I is calculated as \$ 4455.07 (0.6981, 0.6329).

PFNPW of project-II is calculated as \$ 5510.38(0.7446, 0.1808).

Since project-II has a greater PFNPW, it is selected for the investment.

Conclusion: This idea is used to determine an equivalent value for the cash flows of an investment project to decide whether it is profitable or not.Since the estimations of future cash flows and the parameters such as interest rate, useful life are involves high degrees of uncertainty, fuzzy logic and its extensions are used widely in economic analysis of investment projects. In this article, present worth analysis is done. Using single- valued pythagorean fuzzy sets to increase the uncertainty taken into account in the economic analysis.

Future work: We suggested to extend the analysis for several fuzzy membership functions such as interval-valued pythagorean fuzzy sets and neutrosophic sets. Also, pythagorean fuzzy sets could be applied to additional capital budgeting methods such as benefit cost ratio, internal rate of return.

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