Plane Gravitational Waves with Wet Dark Energy in Bimetric Relativity

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Abstract: In this paper, $Z = \frac{1}{\sqrt{2}}(y+z)$- type plane gravitational wave is studied with source Wet Dark Energy in Rosen's Bimetric theory of relativity. It is shown that there is nil contribution of Wet Dark Energy in this theory. Only vacuum model can be constructed.

Keywords: Plane gravitational waves, background metric, Cosmological Constant,Wet Dark Energy, Bimetric Relativity.

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1. Introduction

It is well known that the cosmological models based on general relativity contain an initial singular state (the big bang) from which universe expands. The singular state can be avoided if the behavior of matter and radiation is described by the quantum theory. Unfortunately nobody has given a way to do this satisfactorily. A satisfactory physical theory should be free from singularities because the presence of singularity means a breakdown of physical laws provided by the theory. Naturally, taking into consideration these singularities in general relativity one looks carefully at the foundation of general relativity and thinks whether modification can be made to improve it. With this motivation (GR), Rosen[8-9] has proposed a modified theory of gravitation within the framework of general relativity, which is called Bimetric Theory of Relativity (BR). In this theory, he has proposed a new formulation of the general relativity by introducing a background Euclidean metric tensor γ_{ij} in addition to the usual Riemannian metric tensor g_{ij} at each point of the four dimensional space-time. With the flat background metric, γ_{ij} the physical content of the theory is the same as that of the general relativity.

Thus, now the corresponding two line elements in a coordinate system x^i are –

$$ds^2 = g_{ij}dx^i dx^j \quad (1.1)$$

$$d\sigma^2 = \gamma_{ij}dx^i dx^j \quad (1.2)$$

Where ds is the interval between two neighboring events as measured by means of a clock and ameasuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable.One can regard it as describing the geometry that would exist if no matter were present.H. Takeno [5] propounded a rigorous discussion of plane gravitational waves, defined various terms by formulating a meaningful mathematical version and obtained numerous results.

Using definition of plane wave, we will use here, Z = $\frac{1}{\sqrt{2}}(y+z)$ type plane gravitational waves by using the line elements,

$$ds^2 = -A\left(dx^2 + dy^2\right) - C\left(dz^2 - dt^2\right) \quad (1.3)$$

The theory of plane gravitational waves have been studied by many investigators,H Takeno [6]; S. N. Pandey [15]; I. Goldman[7];R.H. Gowdy[11]; H. Bondi, et.al.[4];C.G.Torre [2]; P. A. Hogan [10];Deo and Ronghe[1];Deo and Suple [12],[13],[14] and they obtained the solutions .

In continuation of this, we will studyZ = $\frac{1}{\sqrt{2}}(y+z)$ type plane gravitational wave with Wet Dark Energy and will observe the result in the context of BimetricTheory of Relativity.

2. Field equations in Bimetric relativity

N. Rosen [1, 2] has proposed the field equations of Bimetric Relativity from Variation Principle as

$$K_i^j = N_i^j - \frac{1}{2}Ng_i^j = -8\pi\kappa T_i^j \quad (2.1)$$

Where $N_i^j = \frac{1}{2}\gamma^{\alpha\beta}\left[g^{hj}g_{hi|\alpha}\right]_{|\beta}$ (2.2)

$$N = N_\alpha^\alpha, \quad \kappa = \sqrt{\frac{g}{\gamma}} \quad (2.3)$$

and $g = \left|g_{ij}\right|$, $\gamma = \left|\gamma_{ij}\right|$ (2.4)

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij} .

3. Z = $\frac{1}{\sqrt{2}}(y+z)$ type plane gravitational wave with wet dark energy

For Z = $\frac{1}{\sqrt{2}}(y+z)$ plane gravitational wave, we have the line element as

$$ds^2 = -A\left(dx^2 + dy^2\right) - C\left(dz^2 - dt^2\right) \quad (3.1)$$

where $A = A(Z), C = C(Z)$ and Z = $\frac{1}{\sqrt{2}}(y+z)$

Corresponding to the equation (3.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -\left(dx^2 + dy^2 + dz^2\right) + dt^2 \quad (3.2)$$

And, T_i^j the energy momentum tensor for Wet Dark Energy is given by

$$T_i^j = T_{i\ wdf}^j = \rho_{wdf} + p_{wdf} u_i u^j - p_{wdf} g_i^j \quad (3.3)$$

together with $g_i^j u_i u^j = 1,\ u_4 u^4 = 1$ where v_i is the flow vector of the fluid having p and ρ as proper pressure and energy density respectively.

In co-moving coordinate system,we have

$$T_1^1 = T_2^2 = T_3^3 = -p_{wdf} \text{ and } T_4^4 = \rho_{wdf}, T_{i\ wdf}^j = 0 \ for\ i \neq j$$

Using equations (2.1) to (2.4) with (3.1) to (3.3), we get the field equations as

$$\left(\frac{C'^2}{C^2} - \frac{C''}{C}\right) = -16\pi\kappa p_{wdf} \quad (3.4)$$

$$\left(\frac{A'^2}{A^2} - \frac{A''}{A}\right) = -16\pi\kappa p_{wdf} \quad (3.5)$$

$$\left(\frac{A'^2}{A^2} - \frac{A''}{A}\right) = 16\pi\kappa \rho_{wdf} \quad (3.6)$$

Where $A' = \frac{\partial A}{\partial Z}$, $A'' = \frac{\partial^2 A}{\partial Z^2}$, $C' = \frac{\partial C}{\partial Z}$, $C'' = \frac{\partial^2 C}{\partial Z^2}$

Using equation (3.4) to (3.6),

we get $p + \rho = 0$ (3.7)

This equation of state is known as false vacuum.In view of reality conditions (Hawking S.W. and Ellis G.F.R) [3]

$p > 0, \rho > 0$ (A)

Where ρ is the energy density of the matter and p is the proper pressure of the matter, implies

$\rho = 0, p = 0$

Equation (3.7) is an analogue of condition [A].

Thus, Equation (3.7) immediately implies that $p = 0, \rho = 0$ i.e. wet dark energy does not exist in Z = $\frac{1}{\sqrt{2}}(y+z)$-type plane gravitational wave in Rosen's Bimetric theory of relativity.

Hence, for vacuum case $\lambda = 0 = \rho$, the field equation reduced to

$$\left(\frac{A'^2}{A^2} - \frac{A''}{A}\right) = 0 \quad (3.8)$$

and

$$\left(\frac{C'^2}{C^2} - \frac{C''}{C}\right) = 0 \quad (3.9)$$

Solving equations (3.8) and (3.9), we get

$$A = R_1\, e^{S_1 Z} \quad (3.10)$$

and

$$C = R_2\, e^{S_2 Z} \quad (3.11)$$

where R_1 , S_1 and R_2 , S_2 are the constants of integration.

Thus substituting the value of (3.10) and (3.11) in (3.1), we get the vacuum line element as

$$ds^2 = -R_1 e^{S_1 Z}\left(dx^2 + dy^2\right) - R_2\, e^{S_2 Z}\left(dz^2 - dt^2\right) \quad (3.12)$$

Thus, it is found that in plane gravitational wave Z = $\frac{1}{\sqrt{2}}(y+z)$, wet dark energy does not survive in Bimetric theory of relativity and only vacuum model can be constructed.

By proper choice of co-ordinates, the metric (3.12) can be transform to

$$ds^2 = -e^{\alpha Z}\left[dx^2 + dy^2 + dz^2 - dt^2\right] \quad (3.13)$$

which is free from singularity at t = 0 and the spatial volume of the model is given by

$$V^3 = (-g)^{\frac{1}{2}} = e^{2\alpha Z} \quad (3.14)$$

This study can further be extended with the introduction of cosmological constant λ in the field equation, which is defined as $N_i^j = \lambda g_i^j$ Thus,Weget, $\left(\frac{A'^2}{A^2} - \frac{A''}{A}\right) = \lambda$ (3.15)

And $\left(\frac{C'^2}{C^2} - \frac{C''}{C}\right) = \lambda$ (3.16)

On solving equation (3.15), we have

$$A = \mathbf{exp}\left[\frac{\lambda Z^2}{2} + EZ + F\right] \quad (3.17)$$

where E and F are constants of integration.

On solving equation (3.16), we obtain

$$C = \mathbf{exp}\left[\frac{\lambda Z^2}{2} + GZ + H\right] \quad (3.18)$$

where G and H are constants of integration.

Thus, substituting the value of A and C [using (3.17)-(3.18)] in the line element (3.1) which reduces to

$$ds^2 = -\mathbf{exp}\left[\frac{\lambda Z^2}{2} + EZ + F\right]\left(dx^2 + dy^2\right) - \mathbf{exp}\left[\frac{\lambda Z^2}{2} + GZ + H\right]\left(dz^2 - dt^2\right) \quad (3.19)$$

Thus, Z = $\frac{1}{\sqrt{2}}(y+z)$ plane gravitational wave exists in Bimetric relativity with or without cosmological constant λ respectively.

4. Conclusions

In this paper, it is shown that, in the study of **Z** = $\frac{1}{\sqrt{2}}(y+z)$-type plane gravitational wave; there is nil contribution of Wet Dark Energy in Bimetric theory of relativity respectively. It is observed that the matter field Wet Dark Energy cannot be a source of gravitational field in the Rosen's Bimetric theory but only vacuum model exists. Hence,Bimetric theory does not help in any way to study gravitational effects of Wet Dark Energy at the early stages of evolution of the universe.

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