# Some fixed theorem for continuous mapping in fuzzy metric spaces

**Upendra Kumar, Assistant professor**
**Department of Mathematics**
**Government Post Graduate College, Obra, Sonbhadra (U.P.)-231219**

## Abstract:

This paper uses weak compatibility and equal continuity to prove popular fixed point theorems in generalised intuitionistic fuzzy metric spaces (FMS). This study aims to demonstrate common fixed point propositions that use rational terms in M-fuzzy metric spaces, while concurrently substantiating our findings. Our findings lead towards a rationalisation of a integer of fixed point theorems found in the body of work on M-FMS.

**Keywords:** Fixed Point Theorems, Fuzzy Sets, Semi Compatibility

## 1. Introduction:

In topology and analysis, Zadeh's (1965) overview of the idea of fuzzy sets is crucial. Numerous writers have since studied fuzzy sets through submissions. Particularly in 1975, Kramosil lead a novel idea of fuzzy metric spaces (FMS). By incessant t-norm, Veeramani (1994) reinterpreted the concept of FMS. This leads to the derivation of numerous fixed point propositions in FMS for different types of mappings. Fuzzy set theory was initially introduced to LPPs by Zimmermann (1978). He took LPPs with ambiguous objectives and restrictions into account. Following the fuzzy choice presented by Bellman and Zadeh (1965), they established that an analogous LPP persists using linear membership functions. The ambiguous solution, or minimalist operator of Zadeh (1965), is used as an example of the DM's dim preference in these ambiguous approaches. Fuzzy set theory remains a accurate theory that attempts to mimic the fuzziness and sketchiness of human thought. The formalization of ambiguity in mathematics, pioneered by Zadeh (1965). Dhage (1992) defined D-metric spaces and established numerous new fixed point theorems in them. The perception of M-FMS stayed first developed by Guangpeng Sun and Kai Yang in 2016. We provide the thought of generalised intuitionistic fuzzy space in this work, which stands a fuzzy space generalisation. This paper uses weak compatibility, and give-and-take continuity to prove popular fixed point theorems in generalised intuitionistic FMS. Our findings broaden, enhance, and vaguely clarify multiple fixed point propositions in M-fuzzy spaces. A generalisation of fuzzy spaces by George and Veeramani

(2015), M-FMS were introduced by Sedghi et al. (2013). They likewise showed mutual fixed point propositions for two mappings below the condition that they are weakly like-minded and R-weakly travelling mappings in comprehensive M-FMS.A research by Park et al. (2008) demonstrated common fixed point propositions for mappings that satisfy certain environments and familiarized the idea of well-matched mapping of type (*) in M-FMS. In this research, we demonstrate that any D*-metric & fuzzy metric, respectively, induces an M-fuzzy metric. We also shown communal fixed point theorems by utilizing compatible mappings of type (*) and rational inequality meeting certain requirements.

**FuzzySetTheory:**

Fuzzy set theory remains a accurate theory that attempts to mimic the fuzziness and sketchiness of human thought. The formalization of ambiguity in mathematics, pioneered by Zadeh (1965) [208]. The alternative is ambiguous logic, which focuses on the certainty with which the outcome falls into a given category rather than the likelihood of its occurrence. As a matter of fact, the nebulous premise is "everything is a question of degree." Thus, affiliation in a nebulous set is not a material of confirmation or denial, nevertheless of degree.

**FuzzyMathematicalProgramming:**

Fuzzy set theory was initially introduced to LPPs by Zimmermann (1978). He took LPPs with ambiguous objectives and restrictions into account. Following the fuzzy choice presented by Bellman and Zadeh (1965), they established that an analogous LPP persists using linear membership functions. The ambiguous solution, or minimalist operator of Zadeh (1965), is used as an example of the DM's dim preference in these ambiguous approaches.

## 2. Preliminaries

**Definition 1:**

For short GIFMS, a 5-tuple (X, Q, H, *, ) is called a generalised intuitionistic FMS. If Q and H stand fuzzy sets on $X^3 \to (0,\infty)$ that satisfy the subsequent requirements, and X is an uninformed set, then * and ⋄ stand incessant t-norms and t-conforms, correspondingly. Every time t, s>0 and x, y, z, a∈X,

i) $Q(x, y, z, t) + H(x, y, z, t) \leq 1$

ii) $\mathrm{Q(x, x, y, t)} > 0, \forall \mathrm{x} \neq \mathrm{y}$

iii) $\mathrm{Q(x, x, y, t)} \leq \mathrm{Q(x, y, z, t)} \forall \mathrm{y} \neq \mathrm{z}$

iv) $Q(x, y, z, t) = 1$ iff $x = y = z$

v) $\mathrm{Q(x, y, z, t)} = \mathrm{Q\{p(x, y, z), t\}}$

vi) $\mathrm{Q(x, a, at)} * \mathrm{Q(a, y, z, s)} \leq \mathrm{Q(x, y, z, t + s)}$

vii) $\mathrm{Q(x, y, z, )} \colon . (0, \infty) \to [0,1]$ is incessant

viii) Q is non-declining value on $\mathrm{R}^{+} \mathrm{lim} t \to \infty \mathrm{Q(x, y, z, t)} = 1$

$$\lim_{t \to 0} Q(x, y, z, t) = 0 \forall \mathrm{x, y, z} \in \mathrm{X, t} > 0$$

ix) $\mathrm{H(x, x, y, t)} < 1, \forall \mathrm{x} \neq \mathrm{y}$

x) $\mathrm{H(x, x, y, t)} \geq \mathrm{H(x, y, z, t)} \forall \mathrm{y} \neq \mathrm{z}$

xi) $H(x, y, z, t) = 0; x = y = z$

xii) $H(x, y, z, t) = H\{p(x, y, z), t\}$

xiii) $\mathrm{H(x, a, at)} \diamond \mathrm{H(a, y, z, s)} \geq \mathrm{H(x, y, z, t + s)}$

xiv) $\mathrm{H(x, y, z, )} \colon . (0, \infty) \to [0,1]$

xv)H a non- increasing value on $\mathrm{R} + \mathrm{lim} t \to \infty \mathrm{H(x, y, z, t)} = 0$

$$\lim_{t \to 0} H(x, y, z, t) = 1 \forall \mathrm{x, y, z} \in \mathrm{X, t} > 0$$

The braces (Q, H) is referred to as a indiscriminate intuitionistic FMS on X in this instance.

**Definition 2:**

Let $(\mathrm{X, Q, H}, *, \diamond)$ stand a comprehensive intuitionistic FMS, then

i) A series {xn} in X stands assumed to stand convergent to x if $\lim_{n \to \infty} Q(x_n, x_n, x, t) = 1$ and $\lim_{n \to \infty} H(x_n, x_n, x, t) = 0$.

ii) A structure {xn} in X stands said to be Cauchy sequence if $\lim_{n,m \to \infty} Q(x_n, x_n, x_m, t)$ =1 and $\lim_{n,m \to \infty} H(x_n, x_n, x_m, t) = 0$ that is, for any $\varepsilon > 0$ and for each $\mathrm{t} > 0$, $\exists\, n_0 \in \mathrm{N}$ s.t. $Q(x_n, x_n, x_m, t) > 1 - \varepsilon$ and $H(x_n, x_n, x_m, t) < \varepsilon$ for $\mathrm{n, m} \geq n_0$.

iii) If all of the Cauchy sequences in X converge, then the generalized intuitionistic FMS (X, Q, H,*,⋄) is considered complete.

**Definition 3:**

On the generalised intuitionistic FMS (X, Q, H,*,⋄), let f & g stand self-maps. If the mappings shuttle at their concurrence point, meaning that fx = gx indicates that fgx = gfx, they are then considered weakly compatible.

**Definition 4:**

Contract $A$ and $S$ remain self-maps continuously a sweeping intuitionistic fuzzy space ( $X, Q, H,*$ ,0).

$$\lim_{n\to\infty} Q(ASx_n, Ax, Ax, t) = 1, \lim_{n\to\infty} Q(SAx_n, Sx, Sx, t) = 1 \text{ and}$$
$$\lim_{n\to\infty} H(ASx_n, Ax, Ax, t) = 0, \lim_{n\to\infty} H(SAx_n, Sx, Sx, t) = 0$$

Whenever there exists a sequence $\{x_n\}$ in $X$ s.t. $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ some $x \in X$.

**Definition 5:**

A generalized intuitionistic FMS (X, Q, H,*,⋄) with two self-maps, A and Son, are considered semi-compatible if

$$\lim_{n\to\infty} Q(ASx_n, Sx, Sx, t) = 1 \text{ and } \lim_{n\to\infty} H(ASx_n, Sx, Sx, t) = 0$$

Whenever $\in$ a sequence $\{x_n\}$ in $X$ s.t. $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ in lieu of some $x \in X$. Reciprocally continuous functions are all continuous functions; however, the opposite is not true.

**Example 6:** Contract X = [2,20] through typical metric and * stand distinct as $a * b = \min\{a, b\}$ and $a \diamond \mathrm{b} = \max\{\mathrm{a}, \mathrm{b}\}$. Define $\mathrm{Q}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}) = \frac{\mathrm{t}}{\mathrm{t}+\mathrm{G}(\mathrm{x},\mathrm{y},\mathrm{z})}$ and $\mathrm{H}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}) = \frac{\mathrm{G}(\mathrm{x},\mathrm{y},\mathrm{z})}{\mathrm{t}+\mathrm{G}(\mathrm{x},\mathrm{y},\mathrm{z})}$.
Where $G(x, y, z) = |x - y| + |y - z| + |z - x|$ is a usual generalized metric. Define

$$Ax = \begin{Bmatrix} 2 \text{ if } x = 2 \\ 3 \text{ if } x > 2 \end{Bmatrix} \text{ and } Sx = \begin{Bmatrix} 2 \text{ if } x = 2 \\ 3 \text{ if } x > 2 \end{Bmatrix}$$

Consider a sequence $\{x_n\}$ in [2,20] s.t. $x_n < 2$ for each n.
Then $\lim_{n\to\infty} \mathrm{Ax_n} = 2, \lim_{n\to\infty} \mathrm{Sxx_n} = 2, \mathrm{Ax_n} \to 2 = \mathrm{A2}$ and $\mathrm{Sxx_n} \to 2 = \mathrm{S2}$ Neither $A$ nor $S$ is reciprocally continuous at 2, A and S are continuous at 2. In fact,

$$\lim_{n\to\infty} Q(ASx_n, A2, A2, t) = \frac{t}{t + |ASx_n - A2| + |ASx_n - A2| + |A2 - A2|} \text{ and}$$

$$\lim_{n\to\infty} Q(ASx_n, A2, A2, t) = \frac{t}{t + 2|ASx_n - A2|}$$

Thus $\lim_{n\to\infty} Q(ASx_n, A2, A2, t) \to 1$ as $n \to \infty$

$$\lim_{n\to\infty} H(ASx_n, A2, A2, t) = \frac{|ASx_n - A2| + |ASx_n - A2| + |A2 - A2|}{t + |ASx_n - A2| + |ASx_n - A2| + |A2 - A2|} \lim_{n\to\infty} H(ASx_n, A2, A2, t)$$

$$= \frac{2|ASx_n - A2|}{t + 2|ASx_n - A2|}$$

Thus $\lim_{n\to\infty} H(ASx_n, A2, A2, t) \to 0$ as $n \to \infty$

This shows that ASx $x_n \to A2$. In like method we acquire SAx $x_n \to S2$.

(A,S) is reciprocally continuous as a result.

## 3. **Four self-maps have a single shared fixed point.**

**Theorem 1:**

In a complete generalized intuitionistic FMS (X,Q,H,*, ⋄), let A, B, S, & T be self-maps. * is a uninterrupted t-norm, and ⋄ stands a continuous t-conform, substantial:

1. $AX \subseteq TX, BX \subseteq SX$
2. (B, T) stands weak well-suited
3. For respectively $x, y, z \in X$ and $t > 0, Q(Ax, By, Bz, t) \geq \Phi(Q(Sx, Ty, Tz, t))$ and $H(Ax, By, Bz, t) \leq \psi(H(Sx, Ty, Tz, t))$, Where $\Phi, \psi: [0,1] \to [0,1]$ is a nonstop function s.t. $\Phi(1) = 1$ and $\psi(0) = 0$ and $\Phi(a) > a, \psi(a) < a$, for each $0 < a < 1$.

If (A, S) is reciprocally continuous and semi compatible, then there is only one mutual fixed point shared by A, B, S, & T.

**Proof:** Contract $x_0 \in X$ stand an indiscriminate point. Then $\in x_1, x_2 \in X$ s.t. $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$. As a result we can construct sequences $\{y_n\}$ and $\{x_n\}$ in x s.t. $y_{2n+1} = Ax_{2n} = Tx_{2n+1}, y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$ for $n = 0,1,2, ....$ By contractive circumstance, we acquire,

$$\left.\begin{aligned}
Q\left(y_{2n+1}, y_{2n+2,} y_{2n+3}, t\right) &= Q(Ax_{2n}, Bx_{2n+1}, Bx_{2n+2}, t)\\
&\geq \Phi\big(Q(Sx_{2n}, Tx_{2n+1}, Tx_{2n+2}, t)\big)\\
&\quad > Q\big(y_{2n}, y_{2n+1}, y_{2n+2}, t\big)\\
H\left(y_{2n+1}, y_{2n+2,} y_{2n+3}, t\right) &= H(Ax_{2n}, Bx_{2n+1}, Bx_{2n+2}, t)\\
&\leq \psi\big(H(Sx_{2n}, Tx_{2n+1}, Tx_{2n+2}, t)\big)\\
&\quad < H\big(y_{2n}, y_{2n+1}, y_{2n+2}, t\big)
\end{aligned}\right\} \ldots.(1)$$

Correspondingly we dismiss obligate $Q\big(y_{2n+2}, y_{2n+3,} y_{2n+4}, t\big) > Q(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)$ and $H(y_{2n+2}, y_{2n+3}, y_{2n+4}, t) < H(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)$

In over-all, we can inscribe

$Q(y_{n+2}, y_{n+1}, y_n, t) > Q(y_{n+1}, y_n, y_{n-1}, t)$ and $H(y_{n+2}, y_{n+1}, y_n, t) < H(y_{n+1}, y_n, y_{n-1}, t)$

Thus $\{Q\big(y_{n+1}, y_n, y_{n-1}, t\big)\}$ is an aggregate sequence and $\{H\big(y_{n+1}, y_n, y_{n-1}, t\big)\}$ stands a declining sequence of optimistic real statistics in [0,1] and inclines to limit $l \leq 1$.

If $l < 1$ before

$Q(y_{n+2}, y_{n+1}, y_n, t) \geq \Phi\big(Q(y_{n+1}, y_n, y_{n-1}, t)\big) H(y_{n+2}, y_{n+1}, y_n, t) \leq$

$\psi\big(H(y_{n+1}, y_n, y_{n-1}, t)\big)$…..(2)

On let n $\to \infty$ we acquire,

$$\left.\begin{aligned}
\lim_{n\to\infty} Q\big(y_{n+2}, y_{n+1}, y_n, t\big) &\geq \Phi\left(\lim_{n\to\infty} Q\big(y_{n+1}, y_n, y_{n-1}, t\big)\right)\\
\lim_{n\to\infty} H\big(y_{n+2}, y_{n+1}, y_n, t\big) &\leq \psi\left(\lim_{n\to\infty} H\big(y_{n+1}, y_n, y_{n-1}, t\big)\right)
\end{aligned}\right\} \ldots..(3)$$

That stands $l \geq \Phi(l) > l$ and $l \leq \psi(l) < l$ a inconsistency. Thus $l = 1$.

At the present for optimistic fraction p,

$$\begin{aligned}
Q\big(y_n, y_{n+p}, y_{n+p}, t\big) &\geq Q(y_n, y_{n+1}, y_{n+1}, t/2) * Q\big(y_{n+1}, y_{n+p}, y_{n+p}, t/2\big)\\
&\geq Q\left(y_n, y_{n+1,}, y_{n+1}, t/p\right) * Q\big(y_{n+1}, y_{n+2}, y_{n+2}, t/p\big) * \ldots *\\
&\qquad Q\left(y_{n+p-1}, y_{n+p,} y_{n+p}, t/p\right)
\end{aligned}$$

$$\left.\begin{aligned}
H\left(y_n, y_{n+p}, y_{n+p}, t\right) &\leq H\big(y_n, y_{n+1}, y_{n+1}, t/2\big) \diamond H\left(y_{n+1}, y_{n+p, y_{n+p}, t/2}\right)\\
&\leq \dot{H}\big(y_n, y_{n+1}, y_{n+1}, t/p\big) \diamond H\big(y_{n+1}, y_{n+2}, y_{n+2}, t/p\big) \diamond \ldots \diamond\\
&\quad H\left(y_{n+p-1}, y_{n+p}, y_{n+p}, t/p\right)
\end{aligned}\right\} \ldots..(4)$$

Attractive limit $\lim_{n\to\infty} Q\left(y_n, y_{n+p}, y_{n+p}, t\right) = 1$ and $\lim_{n\to\infty} H\left(y_n, y_{n+p}, y_{n+p}, t\right) = 0$

$\lim_{n\to\infty} Q\left(y_n, y_{n+p}, y_{n+p}, t\right) \geq 1 * 1 * \ldots * 1 = 1$ and $\lim_{n\to\infty} H\left(y_n, y_{n+p}, y_{n+p}, t\right) \leq 0 \diamond 0 \ldots \nabla 0 = 0$

Thus $\{y_n\}$ stands a Cauchy sequence in X. Subsequently X is comprehensive $y_n \to u$ in X.

That stands, $\{Ax_{2n}\}, \{Tx_{2n+1}\}, \{Bx_{2n+1}\}, \{Sx_{2n+2}\}$ also converges to u in X.

Thus $\lim_{n\to\infty} Sx_{2n} = u$and $\lim_{n\to\infty} Ax_{2n} = u$.

$\lim_{n\to\infty} ASx_{2n} = Au, \lim_{n\to\infty} SAx_{2n} = Su$ and

$\lim_{n\to\infty} Q(ASx_{2n}, Su, Su, t) = 1, \lim_{n\to\infty} H(ASx_{2n}, Su, Su, t) = 0$…..(5)

Thus $Au = Su$.

Currently we determination expression that $Au = u$. Suppose $Au \neq u$. Next, under contractive conditions, we get

$Q(Au, Bx_{2n+1}, Bx_{2n+1}, t) \geq \left(Q(Su, Tx_{2n+1}, Tx_{2n+1}, t)\right)$ and $H(Au, Bx_{2n+1}, Bx_{2n+1}, t) \leq \psi\left(H(Su^{Tx} Tx_{2n+1}, Tx_{2n+1}, t)\right)$

Letting $n \to \infty$,Next, under contractive conditions, we get

$Q(Au, u, u, t) \geq \Phi(Q(Su, u, u, t)) = \Phi(Q(Au, u, u, t)) > Q(Au, u, u, t)$ and

$H(Au, u, u, t) \leq \psi(H(Su, u, u, t)) = \psi(H(Au, u, u, t)) < H(Au, u, u, t)$ a inconsistency. Thus $Au = u = Su$.

Now $AX \subseteq TX$, then $\in$ a $w \in X$ s.t. $u = Au = Tw$. Then by replacing $x = x_{2n}$ and $y = z = w$,we attain:

$$Q(Ax_{2n}, Bw, Bw, t) \geq \Phi\left(Q(Sx_{2n}, Tw, Tw, t)\right) \text{ and } H(Axx_{2n}, Bw, Bw, t) \leq \psi\left(H(Sx_{2n}, Tw, Tw, t)\right) \quad \ldots.(6)$$

Taking limit $n \to \infty$ we get,

$Q(u, Bw, Bw, t) \geq \Phi(Q(u, Tw, Tw, t)) = \Phi(Q(u, u, u, t)) = \Phi(1) = 1$….(7)

$H(u, Bw, Bw, t) \leq \psi(H(u, Tw, Tw, t)) = \psi(H(u, u, u, t)) = \psi(0) = 0$..(8)

Thus $u = Bw = Tw$.Next, under contractive conditions, we get

Also weak compatibility of (B, T ) implies $TBw = BTw$. Thus $Tu = Bu$.

Now we claim that $Au = Bu$. If not,

$Q(Au, Bu, Bu, t) \geq \Phi(Q(Su, Tu, Tu, t))$ and $H(Au, Bu, Bu, t) \leq \psi(H(Su, Tu, Tu, t))$

$$Q(u, Bu, Bu, t) \geq \Phi(Q(u, Bu, Bu, t)) > Q(u, Bu, Bu, t) \text{ and } H(u, Bu, Bu, t) \leq \psi(H(u, Bu, Bu, t)) < H(u, Bu, Bu, t) \quad \ldots.(9)$$

a contradiction. Thus Au = Bu and hence Au = Bu = Tu = Su = u. Accept u, v are dualistic different common fixed arguments of A, B, S & T in order to demonstrate the uniqueness. Next:

$Q(Au, Bv, Bv, t) \geq \Phi(Q(Su, Tv, Tv, t))$

$Q(u, v, v, t) \geq \Phi(Q(u, v, v, t)) > Q(u, v, v, t)$ and

$H(Au, Bv, Bv, t) \leq \psi(H(Su, Tv, Tv, t))$

$H(u, v, v, t) \leq \psi(H(u, v, v, t)) < H(u, v, v, t)$….(10)

a illogicality .

Hence u = v.

The following corollaries result from the aforementioned theorem.

**Theorem 2:**

Assume that the H-FMS (O, Q,*) is complete. Assume that L: O⟶O is a fuzzy contractive mapping where k is contractive continuous, meaning that k subsists within [0,1[ so that

$$\frac{1}{Q(\mathcal{L}\kappa, \mathcal{L}\omega, t)} - 1 \leq k\left(\frac{1}{Q(\kappa, \omega, t)} - 1\right) \ldots (11)$$

$\forall \kappa, \omega$ in $\mathcal{O}$ and $t > 0$. Then, $\mathcal{L}$ devises a unique fixed point $\kappa^*$. After that, $\{\mathcal{L}^n\kappa\}$ is the only stable point for L. Moreover, the sequence$\{\mathcal{L}^n\kappa\}$ converges to κ* for any κ∈O.

**Proof.** Let $\kappa$ in $\mathcal{O}$&$\kappa_n = \mathcal{L}^n\kappa(n \in \mathbb{N})$. Let $t > 0$&$n \in \mathbb{N}$. By disparity (11), we attain

$$\frac{1}{Q(\kappa_{n+1}, \kappa_{n+2}, t)} - 1 \leq k\left(\frac{1}{Q(\kappa_n, \kappa_{n+1}, t)} - 1\right) \ldots (12)$$

$\forall\ t > 0$&$\forall\ n$ in $\mathbb{N}$, which realize that

$$\lim_{n\to\infty} \mathbb{Q}(\kappa_n, \kappa_{n+1}, t) = 1 \ldots (13)$$

$\forall\ t > 0$. At the present, to demonstrate that $\{\kappa_n\}_n$ stands a Cauchy sequence, we accept to different. Since $t \mapsto \mathbb{Q}(\kappa, \omega, t)$ stands a nondecreasing

$\varepsilon \in (0,1)$ and $\xi > 0$

s.t. $p \in \mathbb{N}$, $n_p(\geq p) < m_p \in \mathbb{N}$

$$Q\left(\kappa_{m_p}, \kappa_{n_p}, t\right) \leq 1 - \varepsilon \ldots (14)$$

for all $t < \xi$. Let $t_0 < min\{\xi, r\}$. By feature of limit (13) and last relative, we dismiss write that $\forall \varepsilon \in (0,1)$; $\forall p \in \mathbb{N}$, $\in n_p(\geq p) < m_p \in \mathbb{N}$ :

$$\mathbb{Q}\left(\kappa_{m_p}, \kappa_{n_p}, t_0\right) \leq 1 - \varepsilon$$

Taking keen on account continuousness of utility $t \mapsto \mathbb{Q}(\kappa, \omega, t)$ and information that $\mathbb{Q}\left(\kappa_{m_p-1}, \kappa_{n_p}, t_0\right) > 1-\varepsilon$, we dismiss indicate $q_0 \in \mathbb{N}$ s.t.

$$Q\left(\kappa_{m_p-1}, \kappa_{n_p}, t_0 - \frac{1}{q_0}\right) > 1-\varepsilon \ldots.(15)$$

By strength of expectations (T4) & (F4) and relatives (12) & (13), it shadows that

$$\begin{aligned} 1-\varepsilon &\geq Q\left(\kappa_{m_p}, \kappa_{n_p}, t_0\right) \\ &\geq \widetilde{Q}\left(\kappa_{m_p}, \kappa_{m_p-1}, 0\right) * (1-\varepsilon) \end{aligned}$$

So, conferring to expectations (T2)-(T3), limit (13), single has

$$\lim_{p\to\infty} \mathbb{Q}\left(\kappa_{m_p}, \kappa_{n_p}, t_0\right) = 1-\varepsilon \ldots..(16)$$

What if that $\forall p_1 \geq 0$, $\in p \geq p_1$ s.t. $Q\left(\kappa_{m_p+1}, \kappa_{n_p+1}, t_0\right) \leq 1-\varepsilon$ means, consuming in mind associations (11) & (16), that the categorization $\{\kappa_n\}_n$ has dualistic subsequences $\left\{\kappa_{n_p}\right\}_p \& \left\{\kappa_{m_p}\right\}_p$ confirming

$$\lim_{p\longrightarrow\infty} Q\left(\kappa_{m_p}, \kappa_{n_p}, t_0\right) = \lim_{p\longrightarrow\infty} Q\left(\kappa_{m_p+1}, \kappa_{n_p+1}, t_0\right) = 1-\varepsilon \ldots (17)$$

We have kept the same notation for the subsequence for simplicity's sake.

Currently, we expect that $\in p_1 \geq 0$ s.t. Q $\left(\kappa_{m_p+1}, \kappa_{n_p+1}, t_0\right) > 1-\varepsilon \forall\, p \geq p_1$. We entitlement that $\lim_p \mathfrak{Q}\left(\kappa_{m_p+1}, \kappa_{n_p+1}, t_0\right) = 1-\varepsilon_\varepsilon$. Assume not, i.e., $\in \alpha > 0$ and dualistic subsequences $\left\{\kappa_{n_p}\right\}_p \& \left\{\kappa_{m_p}\right\}_p$ authenticating

$$Q\left(\kappa_{m_p+1}, \kappa_{n_p+1}, t_0\right) > \alpha + (1-\varepsilon) \ldots..(18)$$

$\forall\, p \in \mathbb{N}$.

Devising $q \in \mathbb{N}$ satisfying $\mathbb{Q}\left(\kappa_{m_p+1}, \kappa_{n_p+1}, t_0 - (1/q)\right) > \alpha + (1-\varepsilon)$, we acquire

$$\begin{aligned} 1-\varepsilon &\geq \mathbb{Q}\left(\kappa_{m_p}, \kappa_{n_p}, t_0\right) \\ &\geq \mathbb{Q}\left(\kappa_{m_p}, \kappa_{m_p+1}, \frac{1}{2q}\right) * \mathbb{Q}\left(\kappa_{m_p+1}, \kappa_{n_p+1}, t_0 - \frac{1}{q}\right) \\ &\quad * \mathbb{Q}\left(\kappa_{n_p+1}, \kappa_{n_p}, \frac{1}{2q}\right) \\ &\geq \widetilde{\mathbb{Q}}\left(\kappa_{m_p}, \kappa_{m_p+1}, 0\right) * [\alpha + (1-\varepsilon)] * \widetilde{\mathbb{Q}}\left(\kappa_{n_p+1}, \kappa_{n_p}, 0\right) \end{aligned}$$

as $p \longrightarrow \infty$.

There is inconsistency here. Next,

$$\lim_{p} \mathbb{Q}\left(\kappa_{m_p+1}, \kappa_{n_p+1}, t_0\right) = 1 - \varepsilon \dots\dots (19)$$

A glaring contradiction with condition (11) is reached by relations (14), (15), and (18). Given that $\{\kappa_n\}_n$ is a Cauchy sequence in the whole FMS O, we can infer that $\kappa^* \in \mathcal{O}$ exists such that's.t.

$$\lim_{n} Q(\kappa_n, \kappa^*, t) = 1 \dots . (20)$$

$\forall\ t > 0$, and thru relative (11), we attain

$$\frac{1}{Q(\mathcal{L}\kappa_n, \mathcal{L}\kappa^*, t)} - 1 \leq k\left(\frac{1}{Q(\kappa_n, \kappa^*, t)} - 1\right) \dots \dots (21)$$

for every n in N and every t > 0. Moving on to limit, keeping in mind limit in (19), it shadows that Q(k*,L*,t) =1, which implies that k* is only fixed point of mapping L in accordance with relation (7) and assumption (F2). This brings about the proof.

**Theorem 3:**

Assume that (O, Q,*) is an entire E-FMS. Given a fuzzy Meir-Keeler type mapping L: O⟶O, it can be expressed as follows: for any ε∈(0,1), δ>0 s.t.

$$\varepsilon - \delta < Q(\kappa, \omega, t) \leq \varepsilon \Longrightarrow \mathbb{Q}(\mathcal{L}\kappa, \mathcal{L}\omega, t) > \varepsilon \dots\dots (22)$$

for every κ, ω in O and every t>0. After that, κ* is the only stable point for L. Moreover, the sequence $\{L^n\}$ converges to κ* for any κ∈O.

**Proof.** Let $\kappa \in \mathcal{O} \& \kappa_n = \mathcal{L}^n \kappa (n \in \mathbb{N})$ and $t > 0$. Visibly, we require

$$\mathbb{Q}(\kappa, \mathcal{L}\kappa, t) - \delta < \mathbb{Q}(\kappa, \mathcal{L}\kappa, t) \leq \mathbb{Q}(\kappa, \mathcal{L}\kappa, t) \dots \dots (23)$$

$\forall\ \delta > 0$, & outstanding to relation (21), we achieve $\mathbb{Q}(\mathcal{L}^2\kappa, \mathcal{L}\kappa, t) > Q(\kappa, \mathcal{L}_\kappa, t)$. Recursively, we attain a categorization $\{Q(\kappa_n, \kappa_{n+1}, t)\}_n$ in [0,1] confirming

$$\mathbb{Q}(\kappa_n, \kappa_{n+1}, t) < \mathbb{Q}(\kappa_{n+1}, \kappa_{n+2}, t) \dots . (24)$$

for every n in N. It's an expanding sequence with bounds. After that, a function u:(0,∞)⟶ exists.[0,1] s.t.

$$\lim_{n \longrightarrow +\infty} \mathbb{Q}(\kappa_n, \kappa_{n+1}, t) = \sup_{n \in \mathbb{N}} \mathbb{Q}(\kappa_n, \kappa_{n+1}, t) = u(t) \dots . (25)$$

for all $t > 0$. We prerogative that $u(t) = 1$, $\forall t > 0$. Expect not, i.e., $\in t_0 > 0$ s.t. $u(t_0) \in (0,1)$. By the limit in (25), $\forall \delta \in \left(0, u(t_0)\right)$, there exists $n_0 \in \mathbb{N}$ s.t.

$$u(t_0) - \delta < Q(\kappa_n, \kappa_{n+1}, t_0) \leq u(t_0) \dots \dots (26)$$

for all $n \geq n_0$, which, thru situation (20), involves that Q $(\kappa_{n+1}, \kappa_{n+2}, t_0) > u(t_0)$. This stands a clear illogicality with (23). Consequently,

$$\lim_{n} \mathbb{Q}(\kappa_n, \kappa_{n+1}, t) = 1 \dots (27)$$

Now, we survey, accurately, the alike lines as in the evidence of Proposition 2 to assume that $\{\kappa_n\}_n$ is a Cauchy sequence in the wide-ranging FMS O, which presume that there subsists κ*∈O s.t.

$$\lim_{n} \mathbb{Q}(\kappa^*, \kappa_n, t) = 1 \dots\dots (28)$$

On the supplementary hand, $\forall n \in \mathbb{N}$& all $\delta \in \left(0, Q(\kappa^*, \kappa_n, t)\right)$, we obligate

$$\mathbb{Q}(\kappa^*, \kappa_n, t) - \delta < \mathbb{Q}(\kappa^*, \kappa_n, t) \leq \mathbb{Q}(\kappa^*, \kappa_n, t) \dots\dots (29)$$

Circumstance (20) promises that

$$1 \geq \mathbb{Q}(\mathcal{L}\kappa^*, \mathcal{L}\kappa_n, t) > Q(\kappa^*, \kappa_n, t) \dots\dots (30)$$

which, thru the boundary in (26), stretches $\lim_n Q(\mathcal{L}\kappa^*, \kappa_n, t) = 1$, and lastly

$$\kappa^* = \mathcal{L}\kappa^* \dots \dots (31)$$

In lieu of the exclusivity, we undertake that there subsists $\omega^*(\neq \kappa^*) \in \mathcal{O}$ s.t. $\omega^* = \mathcal{L}\omega^*$. It is strong that for all $\delta \in \left(0, \mathbb{Q}(\kappa^*, \omega^*, t)\right), \mathbb{Q}(\kappa^*, \omega^*, t) - \delta < Q(\kappa^*, \omega^*, t) \leq$ Q $(\kappa^*, \omega^*, t)$.

Later, by (20), $\mathbb{Q}(\mathcal{L}\kappa^*, \mathcal{L}\omega^*, t) > Q(\kappa^*, \omega^*, t)$ or $Q(\kappa^*, \omega^*, t) > \mathbb{Q}(\kappa^*, \omega^*, t)$, a contradiction, and this realizes the impermeable.

## 4. **Application**

This section's goal is to provide an illustration of an integral equation's solution, which may be found by applying Theorem 3. We direct the bibliophile to, where the novelists offer a shared explanation for a organization of dualistic integral equations, for such integral equations.

Examine the integral equation.

$$\kappa(r) = g(r) + \int_0^r F\big(r, s, \kappa(s)\big)\mathrm{d}s, \quad \text{for all } r \in [0, I], I > 0 \dots\dots (32)$$

and the Banach space C([0,I],R) that is furnished with the supremum norm for all nonstop functions distinct on [0,I].

$$\| \kappa \| = \sup_{r \in [0,1]} |\kappa(r)|, \quad \kappa \in C([0, I], \mathbb{R}) \dots . (33)$$

with persuaded metric

$$d(\kappa, \omega) = \sup_{r \in [0,I]} |\kappa(r) - \omega(r)| \dots . (34)$$

Currently, visualize the FMS using product t-norm as

$$\mathbb{Q}(\kappa, \omega, t) = \frac{t}{t + d(\kappa, \omega)}, \quad \text{for all } \kappa, \omega \in C([0, I], \mathbb{R}), t > 0 \ldots. (35)$$

George and Veeramani claim that the topologies of the standard FMS and the associated metric space are the same. Thus, the FMS described in (30) is finished.

**Theorem 4:**

Deliberate the integral operator $\mathcal{L}$ on $C([0, I], \mathbb{R})$ as

$$\mathcal{L}\kappa(r) = g(r) + \int_0^r F(r, s, \kappa(s))\mathrm{d}s \ldots.. (36)$$

Assume that $\in f: [0, I] \times [0, I] \longrightarrow [0, \infty)$ s.t. $f \in L^1([0, I], \mathbb{R})$ and suppose that $F$ gratifies the subsequent circumstance:

$$|F(s, r, \kappa(r)) - F(s, r, \omega(r))| \leq f(r, s)|\kappa(s) - \omega(s)| \ldots. (37)$$

$\forall \kappa, \omega \in C([0, I], \mathbb{R})$ and in lieu of all $r, s \in [0, I]$ everyplace

$$\sup_{r \in [0,1]} \int_0^r f(r, s)\mathrm{d}s \leq k < 1 \ldots. (38)$$

Afterwards, there is only one solution to the integral equation (35).

**Proof.** Let $\kappa, \omega \in C([0, I], \mathbb{R})$ and deliberate

$$\begin{aligned} &| \mathcal{L}\kappa(r) - \mathcal{L}\omega(r) | \\ &\quad \leq \int_0^r |F(r, s, \kappa(s)) - F(r, s, \omega(s))|\mathrm{d}s \\ &\quad \leq d(\kappa, \omega) \int_0^r f(r, s)\mathrm{d}s \\ &\quad \leq kd(\kappa, \omega) \end{aligned}$$

Thus,

$$d(\mathcal{L}\kappa, \mathcal{L}\omega) \leq kd(\kappa, \omega) \ldots.. (39)$$

(36) allows us to write

$$\frac{1}{Q(\kappa,\omega,t)}-1=\frac{d(\kappa,\omega)}{t}$$
$$\frac{1}{Q(\mathcal{L}\kappa,\mathcal{L}\omega,t)}-1=\frac{t+d(\mathcal{L}\kappa,\mathcal{L}\omega)-t}{t}$$
$$\leq k\frac{d(\kappa,\omega)}{t}$$
$$\leq k\left(\frac{1}{Q(\kappa,\omega,t)}-1\right)$$

Theorem 4's criteria are all met, hence (39) has a single solution.

**Theorem 5:**

Let $(X,M,*)$ be a comprehensive $M$-FMS through $t*t\geq t\forall t\in[0,1]$ and circumstance (FM-6). Let $A,B,S,T$ and $P$ be mappings as of $X$ into himself s.t.

(i) $P(X)\subset AB(X)$ and $P(X)\subset ST(X)$,

(ii) ∈s a integer $k\in(0,1)$ s.t.

$$M(Px,Py,Py,kt)\geq M(ABx,Px,Px,t)*M(Px,STy,STy,t)*M(ABx,STy,STy,t)*\frac{M(Px,ABx,ABx,t)*M(Px,STy,STy,t)}{M(STy,ABx,ABx,t)}*M(ABx,Py,Py,(3-\alpha)t) \quad \text{....(40)}$$

$\forall x,y\in X,\alpha\in(0,3)\&t>0$,

(iii) $PB=BP,PT=TP,AB=BA\&ST=TS$,

(iv) A and B are never-ending.

(v) the braces $P,AB$ are companionable of category $(*)$,

(vi) $(x,STx,STx,t)\geq M(x,ABxABx,t)\forall x\in X;t>0$.

In X, A, B, S, T, and P then share a single fixed point.

**Proof:** Since $P(X)\subset AB(X)$, in lieu of ant $x_0\in X$, we dismiss indicate a point $x_0\in X$ s.t. $Px_0=ABx_1$. Since $P(X)\subset ST(X)$, in lieu of this fact $x_1$, we dismiss indicate a opinion $x_2\in X$

s.t. $Px_1 = STx_2$. Thus by initiation, we dismiss outline a classification $y_n \in X$ as shadows: $y_{2n} = Px_{2n} = ABx_{2n+1}$ and $y_{2n+1} = Px_{2n+1} = STx_{2n+1}$ for $n = 1,2,....$ By (ii), for all $t > 0$ and $\alpha = 2 - q$ with $q \in (0,2)$, we have

$$M(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) = M(Px_{2n+1}, Px_{2n+2}, Px_{2n+2}, kt)$$
$$\geq M(y_{2n+1}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n}, y_{2n+1}, y_{2n+1}, t) *$$
$$\frac{M(y_{2n+1}), y_{2n}, y_{2n}, t) * M(y_{2n+1}, y_{2n+1}, y_{2n+1}, t)}{M(y_{2n+1}, y_{2n}, y_{2n}, t)} * M(y_{2n}, y_{2n+2}, y_{2n+2}, (1+q)t) \quad 41)$$
$$M(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n}, y_{2n+2}, y_{2n+2}, (1+q)t)$$
$$\geq M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, y_{2n+2}, qt)$$
$$\geq M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, y_{2n+2}, t)$$

as $q \to 1$. Since $*$ is continuous and $M(x, y, z, *)$ is unceasing, authorizing $q \to 1$ in upstairs equation, we acquire

$$M(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \ldots (42)$$

Correspondingly, we require

$$M(y_{2n+2}, y_{2n+3}, y_{2n+3}, kt) \ldots (43)$$
151

Thus from (42) and (43), it follows that

$$M(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, y_{n+1}, t) * M(y_{n+1}, y_{n+2}, y_{n+2}, t) \ldots (44)$$

for $n = 1,2, ...$ and then for positive integers $n$ and $p$,

$$M(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, y_{n+1}, t) * M(y_{n+1}, y_{n+2}, y_{n+2}, t/k^p) \ldots (45)$$

Thus, since $M(y_{n+1}, y_{n+1}, y_{n+1}, t/k^p) \to 1$ as $p \to \infty$ we have

$$M(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, y_{n+1}, t) \ldots .(46)$$

Hence proved

## 5. Conclusion:

This study aims to demonstrate common fixed point propositions that use rational terms in M-FMS, while concurrently substantiating our findings. Our findings lead towards a rationalization of a number of fixed point theorems found in body of work on M-FMS.

## 6. References:


1. Dhage. B.C., "Generalized metric spaces and mappings with fixed point", Bull. Calcutta Math. Soc., 84(4), 1992, 329-336.
2. George. A and Veeramani. P , " On Some results in fuzzy metric spaces", Fuzzy sets and Systems,64(1994), 395 – 399.
3. Hu. X-Qi, Luo. Q "Coupled coincidence point theorem for contractions in generalized fuzzy metric spaces", Fixed point theory Appl, 2012, 196 (2012).
4. Kramosil. O and Michalek. J, " Fuzzy metric and statistical metric spaces", Kybernetics,11(1975) $330 - 334$.
5. Mohiuddine. SA, Sevli. H, "Stability of Pexiderized quadratic functional equation in intuitionstic fuzzy normed space", journal of computer Appl. Math, 2011, 2137-2146.
6. Sun. G, Yang. K. "Generalized fuzzy metric spaces with Properties", Res.J. Appl. Sci.2, 2010, 673-678.
7. Zadeh L.A., "Fuzzy sets", Inform. and Control, 8 (1965), 338- 353.
8. S. S. Chang, Y. J. Cho, B. S. Lee, J. S. Jung and S. M. Kang, Coincidence point and minimization theorems in fuzzy metric spaces, Fuzzy Sets and Systems 88 (1997) 119-128.
9. M. Grabiec, Fixed point in fuzzy metric spaces, Fuzzy Sets and Systems 27 (1988) 385-389.
10. J. S. Jung, Y. J. Cho and J. K. Kim, Minimization theorems for fixed point theorems in fuzzy metric spaces and applications, Fuzzy Sets and Systems 61 (1994) 199-207.
11. O. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetica 11 (1975) 336-344.
12. J. H. Park, J. S. Park and Y. C. Kwun, A common fixed point theorem in the intuitionistic fuzzy metric spacesIn L. Jiao et al. (Eds.), Advances in natural computation data mining Xian: Xidian University (2006) 293-300.
13. J. H. Park, J. S. Park, and Y. C. Kwun, Fixed points in M-fuzzy metric spaces, Fuzzy Optim. Decis. Mak. 7 (2008) 305-315.
14. S. Sedghi and N. Shobe, Fixed Point Theorem inM-Fuzzy Metric Spaces with property (E), Advances in Fuzzy Mathematics. 1 (2006) 55-65.
15. M. A. Erceg, "Metric spaces in fuzzy set theory," Journal of Mathematical Analysis and

Applications, vol. 69, no. 1, pp. 205-230, 1979.

16. V. Gregori and S. Romaguera, "Some properties of fuzzy metric spaces," Fuzzy Sets and Systems, vol. 115, no. 3, pp. 485-489, 2000.
17. V. Gregori and A. Sapena, "On fixed-point theorems in fuzzy metric spaces," Fuzzy Sets and Systems, vol. 125, no. 2, pp. 245-252, 2002.
18. D. Gopal and C. Vetro, "Some new fixed point theorem in fuzzy mertric spaces," Iranian Journal of Fuzzy Systems, vol. 11, no. 3, pp. 95-107, 2014.
19. D. Gopal, M. Imdad, C. Vetro, and M. Hasan, "Fixed point theory for cyclic weak $\phi$-contraction in fuzzy metric space," Journal of Nonlinear Analysis and Application, vol. 2012, 11 pages, 2012.
20. V. Gupta, R. K. Saini, and M. Verma, "Fixed point theorem by altering distance technique in complete fuzzy metric spaces," International Journal of Computer Aided Engineering and Technology, vol. 13, no. 4, pp. 437-447, 2020.
21. D. Mihet, "A Banach contraction theorem in fuzzy metric spaces," Fuzzy Sets and Systems, vol. 144, pp. 431-439, 2004.
22. P. Borisut, P. Kumam, V. Gupta, and N. Mani, "Generalized $(\psi, \alpha, \beta)$-weak contractions for initial value problems," Mathematics, vol. 7, no. 3, p. 266, 2019.